Math 31 - Fall 2021 - Week 2 - Discussion 2

1. Describe the three elementary row operations for matrices.

2. Convert the system of linear equations

$$\begin{aligned} x - 3z &= 8 \\ 2x + 2y + 9z &= 7 \\ y + 5z &= -2 \end{aligned}$$

into an augmented matrix.

3. Convert the augmented matrix

$$\left(\begin{array}{ccc|c} 3 & 7 & 7 & 6 \\ 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \end{array}\right)$$

into a system of linear equations.

4. What does it mean for a system of linear equations to be consistent? Determine if the systems of equations associated with the following augmented matrices are consistent and, if so, determine if their solutions are unique.

   (a)
$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

   (b)
$$\left(\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 15 \end{array}\right)$$

   (c)
$$\left(\begin{array}{ccc|c} 1 & -6 & 2 & -1 \\ 0 & 1 & 1 & 2 \end{array}\right)$$

5. Solve the system of linear equations

$$\begin{aligned} x - 3y + 4z &= -4 \\ 2x - 7y + 7z &= -8 \\ -4x + 6y - z &= 7 \end{aligned}$$

by converting it to an augmented matrix. This is the same system as in the previous worksheet. Which method seems more efficient?

1/ interchange
2/ scale
3/ replace w/ sum

2/ $\left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \\ x & y & z & \end{array}\right)$

3/ $3x + 7y + 7z = 6$

$y + 4z = -5$

$x + 3y + 5z = -2$

4/ consistent: (, ,) Solution

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4/ a / has sol and unique

b/ $b(1) = 15$: ∴

no solution

not consistent

c/ last row: $y + z = 2$

$$\boxed{y = 2 - z}$$

substitute to first row:

$$x - 6(2-z) + 2z = -1$$

$$x - 12 + 6z + 2z = -1$$

$$\boxed{x = -8z + 11}$$

Any z works : consistent

NOT unique

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$$\left(\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 2 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 2 & -7 & 7 & -8 \\ -4 & 6 & -1 & 7 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -1 & -1 & 0 \\ -4 & 6 & -1 & 7 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -1 & -1 & 0 \\ 0 & -6 & 15 & -9 \end{array}\right)$$

$R2 \leftrightarrow -2R_1 + R_2$ $\qquad$ $R3 \leftrightarrow 4R_1 + R_3$

$$\left(\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & 5 & -3 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 7 & -3 \end{array}\right)$$

$R3 \leftrightarrow -2R_2 + R_3$

$-2(-1) + 5$ $\qquad$ $-2 \cdot 0 + -3$

$-y - z = 0 \quad y = -z.$

$7z = -3$

$z = \frac{-3}{7}$

$y = -z$

$y = \frac{+3}{7}$