Applied Linear Algebra - MATH 31
UC Riverside - Fall 2021 - Practice Midterm Exam

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(1) Let A be the matrix below and b be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

(1a) Write the reduced row echelon form of A.
(1b) Write the general solution of the system $Ax = b$.
(1c) Write the general solution of the system $Ax = 0$.

$[A \mid b]$ → row reduced

(2) Find the matrix representing the linear transformations described below.
(2a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the rotation by an angle of 90 degrees counterclockwise.
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$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(3a) Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$?
(3b) Is T one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

(4a) Find $T(u)$.
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(5a) Find AB.
(5b) Find A^{-1}

$$\left(\begin{array}{cccc|c} 1 & -4 & 8 & 1 & 6 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{4R_2 + R_1 \to R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 13 & 14 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{-3R_3 + R_2 \to R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 13 & 14 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{-13R_3 + R_1 \to R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$x_1 = 1$

$x_4 = 1$

$x_2 - 2x_3 = -1$

$x_2 = 2x_3 - 1$, x_3 free

$\left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ is reduced echelon of A

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2x_3 - 1 \\ x_3 \\ 1 \end{bmatrix}$$

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1/c/ Row operations : $[A \mid 0]$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = 0$

$x_4 = 0$

row reduced echelon

$x_2 - 2x_3 = 0$

$x_2 = 2x_3$, x_3 free

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

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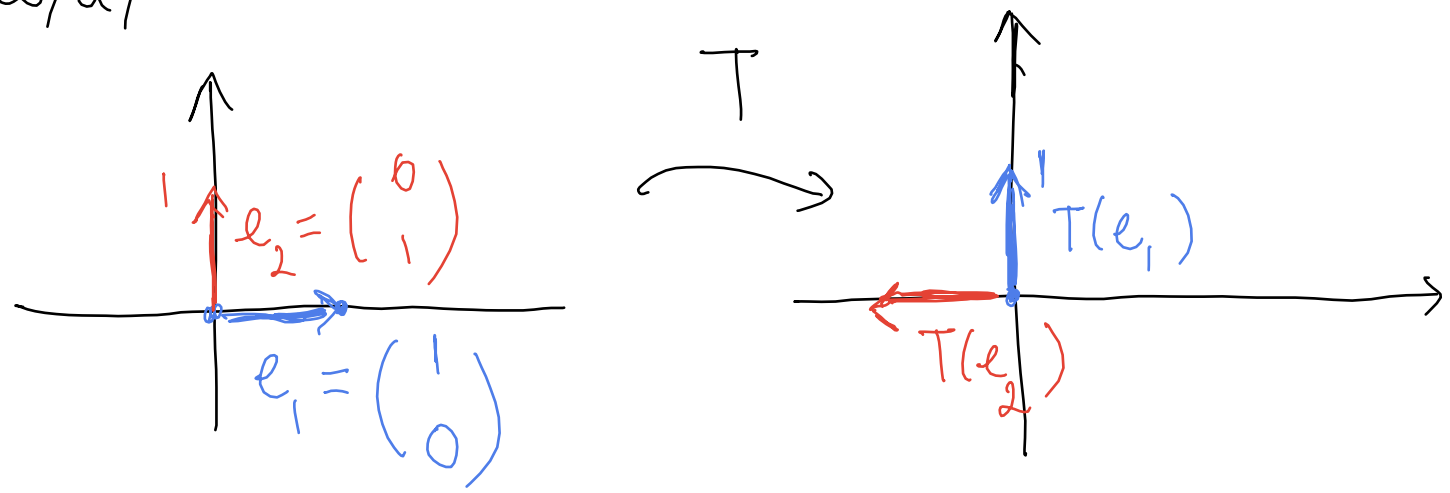
2/a/



$$T(e_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$T(e_2) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Matrix representation of T is $\begin{pmatrix} Te_1 & Te_2 \end{pmatrix}$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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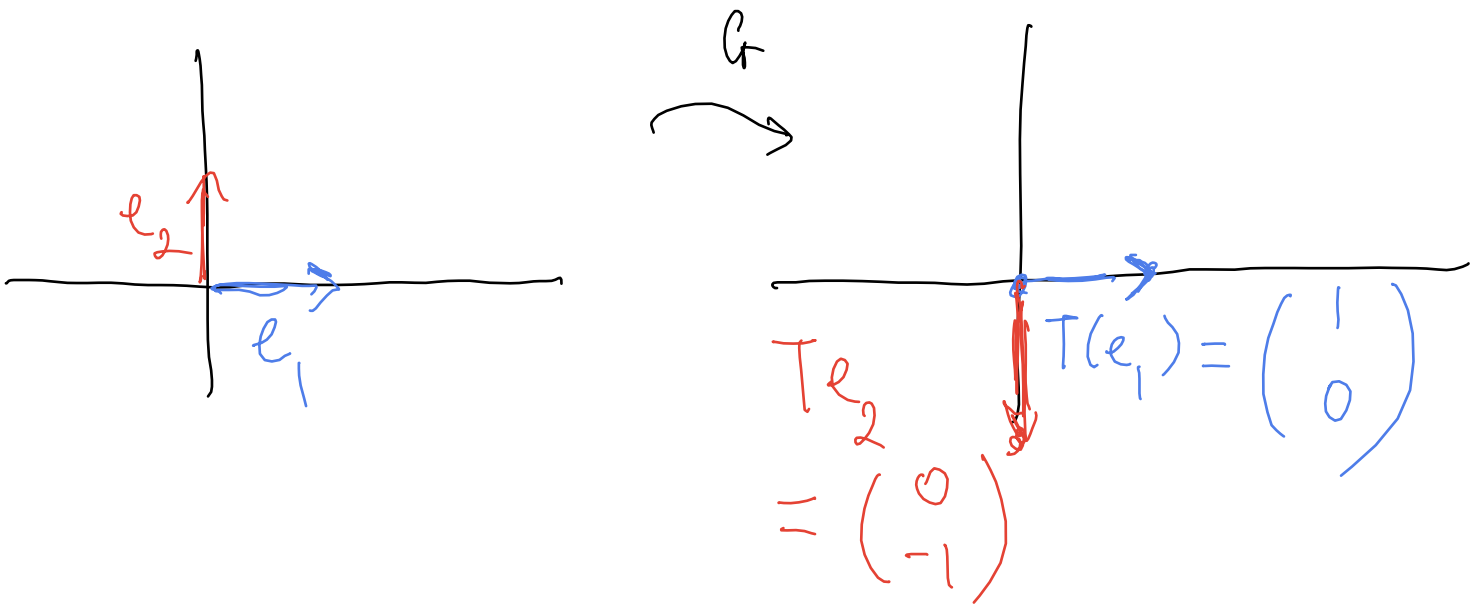

$G_T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

↑ matrix representation of G_T.

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Theorem 12: T onto $\mathbb{R}^3$ if and only if column of A span $\mathbb{R}^3$.

$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}-4\\1\\0\end{pmatrix}, \begin{pmatrix}8\\-2\\0\end{pmatrix}, \begin{pmatrix}1\\3\\1\end{pmatrix}$$

4 vectors in $\mathbb{R}^3$ span $\mathbb{R}^3$?

Theorem 8: column of A will span $\mathbb{R}^3$ if it has 3 pivots.

$$A = \begin{pmatrix}1 & -4 & 8 & 1\\0 & 1 & -2 & 3\\0 & 0 & 0 & 1\end{pmatrix}$$

has 3 pivots $\Rightarrow$ span $\mathbb{R}^3$. $\Rightarrow$ onto.

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Theorem 12 (part 2):

T one-to-one if and only if column of A linearly independent in $\mathbb{R}^3$.

check for 1-to-1 same as check for linearly independence.

Theorem 8: $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}-4\\1\\0\end{pmatrix}, \begin{pmatrix}8\\-2\\0\end{pmatrix}, \begin{pmatrix}1\\3\\1\end{pmatrix}$

4 vectors in $\mathbb{R}^3$, and $4 > 3$.

then they are linearly dependent.

$\Rightarrow$ NOT independence $\Rightarrow$ NOT one to one.

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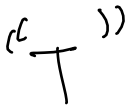

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4/ $Tu = Au = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ -9 \end{pmatrix}$

$3 \cdot 2 + 5(-1) = 1$

$-1 \cdot 2 + 7(-1) = -9$

b/ $Tx = b : Ax = b.$

$$\left(\begin{array}{cc|c} 1 & -3 & 3 \\ 3 & 5 & 2 \\ -1 & 7 & -5 \end{array}\right) \xrightarrow{3R_3 + R_2 \to R_2} \left(\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 26 & -13 \\ -1 & 7 & -5 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & -2 & 1 \\ -1 & 7 & -5 \end{array}\right) \xrightarrow{R_1 + R_3 \to R_3} \left(\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & -2 & 1 \\ 0 & 4 & -2 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

$-2x_2 = 1 \Rightarrow x_2 = -\frac{1}{2}$

$x_1 - 3x_2 = 3$

$x_1 = 3 + 3x_2$

$= 3 + 3\left(-\frac{1}{2}\right) = \frac{3}{2}$

$\vec{x} = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}$

4c/ $\vec{c} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ is $\vec{c}$ in range of T?

Can you find an $\vec{x}$ $A\vec{x} = \vec{c}$ solution?

$$\left(\begin{array}{ccc|c} 1 & -3 & & 3 \\ 3 & 5 & & 2 \\ -1 & 7 & & 5 \end{array}\right)$$

not possible $\rightarrow$ no $\vec{x}$.

c is NOT in the range

$$\rightarrow \left(\begin{array}{ccc|c} 1 & -3 & & 3 \\ 0 & 26 & & 17 \\ -1 & 7 & & 5 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & -3 & & 3 \\ 0 & 26 & & 17 \\ 0 & 4 & & 8 \end{array}\right)$$

$R_1 + R_3 \rightarrow R_3$

$26x_2 = 17$

$x_2 = \frac{17}{26}$

$4x_2 = 8$

$x_2 = 2$