Khoi Vo
MATH_046_001_21F

Assignment 3.1_Euler_Method due 11/06/2021 at 11:59pm PDT

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1. (1 point)

Let $y(t)$ be the solution to $\dot{y} = 8te^{-y}$ satisfying $y(0) = 3$.

(a) Use Euler's Method with time step $h = 0.2$ to approximate $y(0.2), y(0.4), \ldots, y(1.0)$.

	k	t_k	y_k
y_0	0	0	3
y_1	1	0.2	___
y_2	2	0.4	___
y_3	3	0.6	___
y_4	4	0.8	___
y_5	5	1.0	___

(b) Use separation of variables to find $y(t)$ exactly.

$y(t) =$ __________

(c) Compute the error in the approximations to $y(0.2)$, $y(0.6)$, and $y(1)$.

$|y(0.2) - y_1| =$ __________

$|y(0.6) - y_3| =$ __________

$|y(1) - y_5| =$ __________

Answer(s) submitted:

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(incorrect)

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3. (1 point) Use Euler's method with the given step size to estimate $y(1.4)$ where $y(x)$ is the solution of the initial-value problem

$$y' = x - xy, \quad y(1) = 1.$$

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1. Estimate $y(1.4)$ with a step size $h = 0.2$.

Answer: $y(1.4) \approx$ __________

2. Estimate $y(1.4)$ with a step size $h = 0.1$.

Answer: $y(1.4) \approx$ __________

Answer(s) submitted:

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(incorrect)

a/

$y_1 = y_0 + (0.2)\cdot F(\quad)$

derivatives

$= 3 + (0.2)\cdot(8\cdot \underset{t}{0} \cdot e^{-0})$

$= 3 + 0 = 3$

$y_2 = y_1 + (0.2)\, F(\underset{t}{0.2}, \underset{y}{3})$

$= 3 + 0.2\left[8\cdot(0.2)\, e^{-3}\right]$

≈ 3.01593

$y_3 = y_2 + (0.2)\, F(\underset{t}{0.4}, \underset{y}{3.01593})$

$= 3.01593 + (0.2)\left[8\cdot(0.4)\cdot e^{-3.01593}\right]$

$= 3.04729.$

b/ $y' = 8te^{-y}$, $y(0)=3$

$\frac{dy}{dt} = 8t\, e^{-y}$

$e^y\, dy = 8t\, dt$

$e^y = 4t^2 + C$

$y = \ln(4t^2 + C)$

$3 = \ln(4\cdot 0^2 + C)$

$C = e^3$

$\underline{y = \ln(4t^2 + e^3)}$

$|y(0.2) - y_1|$

$|\ln(4(0.2)^2 + e^3) - 3|$

$= 0.007$

$\underline{\underline{16 \; p69}}$ / $y' = \dfrac{y^2 + 2xy}{x^2}$ | write $\left(\dfrac{y}{x}\right)$

$$y' = \underbrace{\frac{y^2}{x^2}} + 2\underbrace{\frac{y}{x}} \qquad \text{Let } u = \frac{y}{\cancel{x}}$$

$$u + u'x = u^2 + 2u \qquad xu = y$$

$$u'x = u^2 + u \qquad y' = (xu)'$$

$$\frac{du}{dx}x = u^2 + u \qquad = x'u + u'x$$

$$\boxed{y' = 1 \cdot u + u'x}$$

deriv w.r.t x

$$\frac{du}{u^2+u} = \frac{1}{x}dx$$

RHS: $\displaystyle\int \frac{1}{x}dx = \ln x$

LHS: $\displaystyle\int \frac{1}{u^2+u}du = \int \frac{1}{u(u+1)}du$

$$= \int \left(\frac{1}{u} - \frac{1}{u+1}\right)du = \ln|u| - \ln|u+1|$$

$$= \ln\left|\frac{u}{u+1}\right|$$

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1. (1 point)

A continuous annuity with withdrawal rate N = $1,400/year and interest rate r = 5

(a) When will the annuity run out of funds if $P_0 = \$23{,}000$?

The annuity runs out after approximately ________ years.

Answer to the nearest whole year.

(b) Which initial deposit P_0 yields a constant balance? $P_0 =$ $ ________

Answer(s) submitted:

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(incorrect)

The instantaneous rate of change of the value of a certain investment (P) is proportional to its value. That is to say $\frac{dP}{dt} = rP$.

If $r = 7$ and $P(0) = 3500$:

$P(t) =$ ________________.

Answer(s) submitted:

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(incorrect)

$$\ln\left|\frac{u}{u+1}\right| = \ln|x|$$

$$\frac{u}{u+1} = x$$

$$\frac{\frac{y}{x}}{\frac{y}{x}+1} = x$$

$$\frac{y}{y+1} = x$$

$$y = (y+1)x$$

$$y = yx + x$$

$$y(1-x) = x$$

$$y = \frac{x}{1-x}$$

1/ Linear First Order: $y' + p(x)y = q(x)$

2/ Separable: $f(x)\,dx = g(y)\,dy$

3/ Homogeneous: find $\boxed{u = \frac{y}{x}}$

4/ Exact:

5/ Bernoulli:

$$M\,dx + N\,dy = 0$$

$$M_y \overset{?}{=} N_x$$

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1. (1 point) A tank contains 100 kg of salt and 2000 L of water. Pure water enters a tank at the rate 6 L/min. The solution is mixed and drains from the tank at the rate 3 L/min.

(a) What is the amount of salt in the tank initially?
amount = ______ (kg)

(b) Find the amount of salt in the tank after 5 hours.
amount = ______ (kg)

(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)
concentration = ______ (kg/L)

Answer(s) submitted:

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-

(incorrect)

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2. (1 point) A tank contains 100 kg of salt and 2000 L of water. A solution of a concentration 0.025 kg of salt per liter enters a tank at the rate 7 L/min. The solution is mixed and drains from the tank at the same rate.

a.) What is the concentration of our solution in the tank initially?

concentration = ______ (kg/L)

b.) Find the amount of salt in the tank after 4.5 hours.

amount = ______ (kg)

c.) Find the concentration of salt in the solution in the tank as time approaches infinity.

concentration = ______ (kg/L)

Answer(s) submitted:

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(incorrect)

2 p.68: $7xy' - 2y = -\frac{x^2}{y^6}$

$\downarrow$ divide by $7x$

$$y' - \frac{2}{7x}y = -\frac{x}{7y^6}$$

Step 1: $y' - \frac{2}{7x}y = 0$

$$\frac{y'}{y} = \frac{2}{7x}$$

$$\ln y = \frac{2}{7}\ln x \Rightarrow \boxed{y = x^{\frac{2}{7}}} \quad y_1$$

Step 2: $y = u \cdot y_1 = u \cdot x^{2/7}$

$$y' = u' x^{2/7} + u \cdot \frac{2}{7} x^{-5/7}$$

$$\left(u' x^{2/7} + u \cdot \frac{2}{7} x^{-5/7}\right) - \frac{2}{7x} \cdot u x^{2/7} = \frac{-x}{7y^6}$$

$$-\frac{2}{7} \cdot x^{-1} u x^{2/7} = \frac{-2}{7} u x^{-5/7}$$

$$u' x^{2/7} = \frac{-x}{7y^6}$$

$$u' x^{2/7} = \frac{-x}{7\left(u x^{2/7}\right)^6}$$

$$u' x^{2/7} = \frac{-x}{7u^6 x^{12/7}}$$

$$\left(u' \cdot 7u^6\right) = \frac{-x}{x^{2/7}\, x^{12/7}} = \frac{-x}{x^2} = \frac{-1}{x}$$

$$\int du\, 7u^6 = \int -\frac{1}{x}\, dx$$

$$u^7 = -\ln x \Rightarrow u = \sqrt[7]{-\ln x}$$

$$y = \sqrt[7]{-\ln x} \cdot x^{2/7}$$