1. (1 point)

Let $y(t)$ be the solution to $\dot{y} = 8te^{-y}$ satisfying $y(0) = 3$.

(a) Use Euler's Method with time step $h = 0.2$ to approximate $y(0.2), y(0.4), ..., y(1.0)$.

k	t_k	y_k
0	0	3
1	0.2	___
2	0.4	___
3	0.6	___
4	0.8	___
5	1.0	___

(b) Use separation of variables to find $y(t)$ exactly.

$y(t)$ = __________

(c) Compute the error in the approximations to $y(0.2)$, $y(0.6)$, and $y(1)$.

$|y(0.2) - y_1| =$ __________

$|y(0.6) - y_3| =$ __________

$|y(1) - y_5| =$ __________

Answer(s) submitted:

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(incorrect)

3. (1 point) Use Euler's method with the given step size to estimate $y(1.4)$ where $y(x)$ is the solution of the initial-value problem

$$y' = x - xy, \quad y(1) = 1.$$

1. Estimate $y(1.4)$ with a step size $h = 0.2$.

Answer: $y(1.4) \approx$ __________

2. Estimate $y(1.4)$ with a step size $h = 0.1$.

Answer: $y(1.4) \approx$ __________

Answer(s) submitted:

-
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(incorrect)

3/p.79 : $\underbrace{14x^2y^3}_{M}dx + \underbrace{21x^2y^2}_{N}dy = 0$

1st step: Check for exactness: $M_y = (14x^2y^3)'_y = 14x^2\cdot 3y^2 = 42x^2y^2$

$N_x = (21x^2y^2)'_x = 21y^2\cdot 2x = 42xy^2$

NOT exact: $42xy^2 \neq 42x^2y^2$

20/p.80: $\underbrace{(y^3-1)e^x}_{M}dx + \underbrace{3y^2(e^x+1)}_{N}dy = 0, \quad y(0)=0$

1st step: exactness? $M_y = (y^3-1)e^x = [e^xy^3 - e^x]'_y = 3y^2e^x$ equal $\to$ EXACT

$N_x = 3y^2(e^x+1) = [3y^2e^x + 3y^2]'_x = 3y^2e^x$

2nd step: $F(x,y) = \int M\,dx = \int (y^3-1)e^x\,dx = (y^3-1)\int e^x\,dx$

$F(x,y) = (y^3-1)e^x + g(y)$

3rd step: find $g(y)$. $[F(x,y)]'_y = N$: compare F_y with N

$[y^3e^x - e^x + g(y)]'_y = N$

$\cancel{3y^2e^x} - 0 + g'(y) = 3y^2(e^x+1) = \cancel{3y^2e^x} + 3y^2$

$\boxed{g'(y)} = 3y^2 \Rightarrow g(y) = \int 3y^2\,dy$

$\boxed{g(y)} = y^3 + C$

4th Step: $F(x,y) = (y^3-1)e^x + y^3 + C = 0$

$y(0)=0$: when $x=0, y=0$: $(0^3-1)e^0 + 0^3 + C = 0$

$-1 + C = 0$

$C = 1.$

$$\boxed{(y^3-1)e^x + y^3 + 1 = 0}$$

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1. (1 point)

A continuous annuity with withdrawal rate N = \$1,400/year and interest rate r = 5

(a) When will the annuity run out of funds if $P_0 = \$23{,}000$?

The annuity runs out after approximately ________ years.
Answer to the nearest whole year.

(b) Which initial deposit P_0 yields a constant balance? P_0 = \$ ________

Answer(s) submitted:

-
-

(incorrect)

The instantaneous rate of change of the value of a certain investment (P) is proportional to its value. That is to say $\frac{dP}{dt} = rP$.

If $r = 7$ and $P(0) = 3500$:

$P(t) =$ ________________.

Answer(s) submitted:

-

(incorrect)

23 p.80/ $\underbrace{(7x+4y)}_{M}dx + \underbrace{(4x+3y)}_{N}dy = 0$

1st step: exact? $M_y = 4 \overset{?}{=} N_x = 4 \Rightarrow$ exact.

2nd step: $F(x,y) = \int M\,dx = \int (7x+4y)\,dx$

$= \frac{7x^2}{2} + 4yx + \underbrace{g(y)}_{\text{constant w.r.t } x}$.

3rd step: find $g(y)$: Compare N with derivative (in y) of F

$$[F(x,y)]'_y = N$$

$$\left[\frac{7x^2}{2} + 4yx + g(y)\right]'_y = N$$

$$0 + 4\cancel{x} + g'(y) = N = 4\cancel{x} + 3y$$

$$g'(y) = 3y \Rightarrow g(y) = \int 3y\,dy = \frac{3y^2}{2} + C$$

$$F(x,y) = \frac{7x^2}{2} + 4yx + \frac{3y^2}{2} + C = 0.$$

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1. (1 point) A tank contains 100 kg of salt and 2000 L of water. Pure water enters a tank at the rate 6 L/min. The solution is mixed and drains from the tank at the rate 3 L/min.

(a) What is the amount of salt in the tank initially?
amount = ________ (kg)

(b) Find the amount of salt in the tank after 5 hours.
amount = ________ (kg)

(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)
concentration = ________ (kg/L)

Answer(s) submitted:

-
-
-

(incorrect)

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2. (1 point) A tank contains 100 kg of salt and 2000 L of water. A solution of a concentration 0.025 kg of salt per liter enters a tank at the rate 7 L/min. The solution is mixed and drains from the tank at the same rate.

a.) What is the concentration of our solution in the tank initially?

concentration = ________ (kg/L)

b.) Find the amount of salt in the tank after 4.5 hours.

amount = ________ (kg)

c.) Find the concentration of salt in the solution in the tank as time approaches infinity.

concentration = ________ (kg/L)

Answer(s) submitted:

-
-
-

(incorrect)

2$\cancel{p}$. 68/ $7xy' - 2y = \dfrac{-x^2}{y^6}$

"y'" must have coefficient 1.

So: $y' - \dfrac{2}{7x} y = -x \cdot y^{-6} \cdot \dfrac{1}{7}$ (*)

- $y' - \dfrac{2}{7x} y = 0$: $\dfrac{y'}{y} = \dfrac{2}{7x} \Rightarrow \ln y = \dfrac{2}{7} \ln x$

$\Rightarrow y = x^{\frac{2}{7}}$

$y_1 = x^{\frac{2}{7}}$ (Step 1)

- Step 2: $\boxed{y = u \cdot y_1 = u \cdot x^{2/7}}$

$y' \underset{\text{product rule}}{=} u' x^{2/7} + \dfrac{2}{7} u x^{-5/7}$

- Step 3: put back original: $u' x^{2/7} + \boxed{\dfrac{2}{7} u x^{-5/7} - \dfrac{2}{7x} \cdot u x^{2/7}} = -xy^{-6} \cdot \dfrac{1}{7}$

$-\dfrac{2}{7x} u x^{2/7} \rightarrow x^{-5/7} \cdot -\dfrac{2}{7} u$

$u' x^{2/7} = -\dfrac{1}{7} x y^{-6}$

$u' = -\dfrac{1}{7} x y^{-6} \cdot x^{-2/7} = -\dfrac{1}{7} x^{5/7} y^{-6}$

$= -\dfrac{1}{7} x^{5/7} (u x^{2/7})^{-6}$