Math 46 - Fall 2021 - Discussion section 002 and 005
Instructor: Prof. Feng Xu
Homework due Oct 22

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$\boxed{u(x) = \left(4\ln x - \frac{1}{5}\right)\cdot x}$

$\frac{1}{y} = \left(4\ln x - \frac{1}{5}\right)x$

$\boxed{y = \frac{1}{\left(4\ln x - \frac{1}{5}\right)x}}$

1 2.4. Transformation of Non Linear Equations into Separable Equations

Exercise 1. A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

6/ $u(x) = (4\ln x + C)\cdot x$

$-\frac{1}{5} = (4\ln 1 + C)\cdot 1$

Use an appropriate substitution to solve the equation

$$xy' + y = 9xy^2,$$

$\Rightarrow \boxed{C = -\frac{1}{5}}$

and find the solution that satisfies $y(1) = -5$.

d/ $u' + \left(-\frac{1}{x}\right)u = 0$

1

$u' = \frac{1}{x}u$

$\frac{u'}{u} = \frac{1}{x}$ take $\int$

$\ln u = \ln x$

$\boxed{u_1 = x}$

2 $u(x) =$ "guessed" $v \cdot u_1$; 1/

2/ $u(x) = v\cdot x$

3/ $u' = v'x + v$

4/ put it back $\Rightarrow v'x + v + \left(-\frac{1}{x}\right)u = 4$

$v'x + v + -\frac{1}{x}(vx) = 4$

$v'x + v - v = 4$

$v'x = 4 \Rightarrow v' = \frac{4}{x}$ integrate $\Rightarrow v = 4\ln x + C$

Exercise 2. A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \quad (*)$$

y'

Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $\boxed{u = y^{1-n}}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

$u = y^{1-2} = y^{-1} = \frac{1}{y}$

$y' + \frac{1}{x}y = -4y^2$

Consider the initial value problem

$n = 2$

$$xy' + y = -4xy^2, \quad y(1) = -5.$$

$\underbrace{\frac{1}{x}}$ $\underbrace{-4}$

(a) This differential equation can be written in the form $(*)$ with $P(x) = ?$, $Q(x) = ?$, and $n = ?$

$u = y^{1-2} = y^{-1}$

(b) The substitution $u = ?$ will transform it into the linear equation $\boxed{\frac{du}{dx} + ?u = ?}$

(c) Using the substitution in part (b), we rewrite the initial condition in terms of x and u: $u(1) = ?$

(d) Now solve the linear equation in part (b), and find the solution that satisfies the initial condition in part (c). $u(x) = ?$

$u' = \frac{du}{dx} = -\frac{1}{y^2} \cdot y'$

(e) Finally, solve for y.

$y(x) =$ ______________

$u = y^{-1} : u' = -1y^{-2} \cdot y'$

$= \boxed{\frac{-y'}{y^2} = u'}$

$\checkmark$

① $\Rightarrow \boxed{\frac{y'}{y^2}} + \frac{1}{x}\frac{y}{y^2} = -4 \checkmark$

② $\Rightarrow -u' + \frac{1}{x}y^{-1} = -4$

③ $\Rightarrow -u' + \left(\frac{1}{x}\right)u = -4$

④ $\Rightarrow \boxed{u' + \left(-\frac{1}{x}\right)u = 4}$

① $y(1) = -5$

② $x = 1, y = -5 \checkmark$

$u(1)$

$x = 1 : u = y^{-1} \swarrow$

$= (-5)^{-1}$

$= \frac{-1}{5} \checkmark$

Exercise 3. A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

Use an appropriate substitution to solve the equation

$$y' - \frac{6}{x}y = \frac{y^4}{x^{16}},$$

and find the solution that satisfies $y(1) = 1$.

2 Exact

Exercise 1. Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function F(x,y) whose differential, $dF(x,y)$ is the left hand side of the differential equation. That is, level curves $F(x,y) = C$ are solutions to the differential equation:

$$(-1x^4 + 2y)dx + (2x + 3y^2)dy = 0$$

First:

$M_y(x,y) =?$, and $N_x(x,y) =?$

If the equation is not exact, enter *not exact,* otherwise enter in $F(x,y)$ here ________________

Exercise 2. Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function F(x,y) whose differential, $dF(x, y)$ is the left hand side of the differential equation. That is, level curves $F(x, y) = C$ are solutions to the differential equation:

$$(2xy^2 + 3y)dx + (2x^2y + 3x)dy = 0$$

First: $M_y(x, y) =$ ______________ , and $N_x(x, y) =$ ______________.

If the equation is not exact, enter *not exact,* otherwise enter in $F(x, y)$ here ______________

Exercise 3. Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function $F(x, y)$ whose differential, $dF(x, y)$ gives the differential equation. That is, level curves $F(x, y) = C$ are solutions to the differential equation:

$$\frac{dy}{dx} = \frac{x^3 - y}{x + 4y^4}$$

First rewrite as

$$M(x, y)\, dx + N(x, y)\, dy = 0$$

where $M(x, y) =$ ______________ ,

and $N(x, y) =$ ______________.

If the equation is not exact, enter *not exact,* otherwise enter in $F(x, y)$ as the solution of the differential equation here ____________________ $= C$.

Exercise 4. Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function $F(x, y)$ whose differential, $dF(x, y)$ is the left hand side of the differential equation.

That is, level curves $F(x, y) = C$ are solutions to the differential equation

$$(-4e^x \sin(y) + 2y)dx + (2x + 4e^x \cos(y))dy = 0$$

First, if this equation has the form $M(x, y)dx + N(x, y)dy = 0$:

$M_y(x, y) =$ ______________ , and $N_x(x, y) =$ ______________.

If the equation is not exact, enter *not exact,* otherwise enter in $F(x, y)$ here ______________

$$y - 3y^2 = (y^6 + x)\frac{dy}{dx}$$

(mult with dx

Exercise 5. The differential equation

$$y - 3y^2 = (y^6 + x)\, y' \qquad (y - 3y^2)\,dx = (y^6 + x)\,dy$$

can be written in differential form:

$$\underbrace{(y - 3y^2)}_{M = y - 3y^2}\,dx + \underbrace{(-y^6 - x)}_{N = -y^6 - x}\,dy = 0$$

$$\boxed{M(x, y)\, dx + N(x, y)\, dy = 0}$$

where

$M(x, y) =$ $y - 3y^2$, and $N(x, y) =$ $-y^6 - x$.

The term $M(x, y)\, dx + N(x, y)\, dy$ becomes an exact differential if the left hand side above is divided by y^2. Integrating that new equation, the solution of the differential equation is ______________ $= C$.

$$\underbrace{\left(\frac{1}{y} - 3\right)}_{M'} dx + \underbrace{\left(-y^4 - \frac{x}{y^2}\right)}_{N} dy = 0$$

$$M_y = -\frac{1}{y^2} \overset{?}{=} N_x = -\frac{1}{y^2}$$

$$F(x, y) = \int M'\,dx = \int \left(\frac{1}{y} - 3\right) dx = \frac{1}{y}x - 3x + g(y)$$

$$\frac{dF}{dy} = \frac{d}{dy}\left[\frac{x}{y} - 3x + g(y)\right] = \frac{-x}{y^2} - 0 + g'(y) = N$$

compare

$$\frac{-x}{y^2} + g'(y) = \frac{-x}{y^2} - y^4$$

$$g'(y) = -y^4 \Rightarrow g = \int -y^4\,dy$$

$$g(y) = \frac{-y^5}{5} + K$$

$$\boxed{F(x, y) = \frac{x}{y} - 3x - \frac{y^5}{5} + K} = 0$$

$$-K$$

$$\frac{x}{y} - 3x - \frac{y^5}{5} = C \quad ✓$$

$$C = -K$$