

Assignment 3.1 Euler Method due 11/06/2021 at 11:59pm PDT

1. (1 point)

Let $y(t)$ be the solution to $\dot{y} = 8te^{-y}$ satisfying $y(0) = 3$.(a) Use Euler's Method with time step $h = 0.2$ to approximate $y(0.2), y(0.4), \dots, y(1.0)$.

k	t_k	y_k
0	0	3
1	0.2	—
2	0.4	—
3	0.6	—
4	0.8	—
5	1.0	—

(b) Use separation of variables to find $y(t)$ exactly. $y(t) =$ _____(c) Compute the error in the approximations to $y(0.2), y(0.6)$, and $y(1)$. $|y(0.2) - y_1| =$ _____ $|y(0.6) - y_3| =$ _____ $|y(1) - y_5| =$ _____

Answer(s) submitted:

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$$\dot{y} = F(t, y)$$

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(incorrect)

3. (1 point) Use Euler's method with the given step size to estimate $y(1.4)$ where $y(x)$ is the solution of the initial-value problem

$$y' = x - xy, \quad y(1) = 1.$$

1. Estimate $y(1.4)$ with a step size $h = 0.2$.Answer: $y(1.4) \approx$ _____2. Estimate $y(1.4)$ with a step size $h = 0.1$.Answer: $y(1.4) \approx$ _____

Answer(s) submitted:

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(incorrect)

$$y_1 = y_0 + h \cdot \underline{y(k=0, t=0)} \quad \text{"y" = 0}$$

$$= 3 + 0.2 * (8.0 e^{-0})$$

$$= 3 + 0.2 * (8 + 0 + 1) = 3$$

$$y_2 = y_1 + \underline{h} \cdot F(t=2, \text{"y"}=3)$$

$$= 3 + 0.2 * (8 + 2 + e^{-3}) = \dots$$

Assignment 4.1_Growth_and_Decay due 11/06/2021 at 11:59pm PDT

1. (1 point)

A continuous annuity with withdrawal rate $N = \$1,400/\text{year}$ and interest rate $r = 5$

(a) When will the annuity run out of funds if $P_0 = \$23,000$?

The annuity runs out after approximately _____ years.

Answer to the nearest whole year.

(b) Which initial deposit P_0 yields a constant balance? $P_0 =$
\$ _____

Answer(s) submitted:

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(incorrect)

The instantaneous rate of change of the value of a certain investment (P) is proportional to its value. That is to say $\frac{dP}{dt} = rP$.

If $r = 7$ and $P(0) = 3500$:

$P(t) =$ _____.

Answer(s) submitted:

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(incorrect)

Assignment 4.2_Cooling_and_Mixing due 11/06/2021 at 11:59pm PDT

1. (1 point) A tank contains 100 kg of salt and 2000 L of water. Pure water enters a tank at the rate 6 L/min. The solution is mixed and drains from the tank at the rate 3 L/min.

(a) What is the amount of salt in the tank initially?

amount = _____ (kg)

(b) Find the amount of salt in the tank after 5 hours.

amount = _____ (kg)

(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)

concentration = _____ (kg/L)

Answer(s) submitted:

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(incorrect)

2. (1 point) A tank contains 100 kg of salt and 2000 L of water. A solution of a concentration 0.025 kg of salt per liter enters a tank at the rate 7 L/min. The solution is mixed and drains from the tank at the same rate.

a.) What is the concentration of our solution in the tank initially?

concentration = _____ (kg/L)

b.) Find the amount of salt in the tank after 4.5 hours.

amount = _____ (kg)

c.) Find the concentration of salt in the solution in the tank as time approaches infinity.

concentration = _____ (kg/L)

Answer(s) submitted:

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(incorrect)

$$y =$$

$$y = \left(\frac{1 - Ke^{2x}}{Ke^{2x} + 1} \right) \cdot x$$

$$2 = \left(\frac{1 - Ke^2}{Ke^2 + 1} \right) \cdot 1$$

$$2(Ke^2 + 1) = 1 - Ke^2$$

$$Ke^2 + 2 = 1$$

$$Ke^2 = -1$$

$$K = \frac{-1}{e^2}$$

$$2/ \underline{x^2} y' = y^2 + xy - x^2, \quad y(1) = 2$$

"Bernoulli"

$$+ \text{If } x^2 = 0 \Rightarrow x = 0: 0 = y^2 + 0 - 0 \Rightarrow \underline{y = 0}.$$

$$* \text{If } x^2 \neq 0: y' = \frac{1}{x^2} y^2 + \frac{1}{x} y - 1$$

$$y' - \frac{1}{x} y = \frac{1}{x^2} y^2 - 1$$

$$* y' - \frac{1}{x} y = 0: y' = \frac{1}{x} y: \frac{y'}{y} = \frac{1}{x}: \ln y = \ln x$$

$$\boxed{y_1 = x}$$

$$* \text{Step 2: } y = u \cdot x \Rightarrow \boxed{y' = u'x + u}$$

$$u'x + u - \frac{1}{x}(ux) = \frac{1}{x^2}(ux)^2 - 1$$

$$u'x + u - u = u^2 - 1 \Rightarrow \boxed{u'x = u^2 - 1}$$

$$\frac{u'}{u^2 - 1} = \frac{1}{x} \Rightarrow \frac{du}{u^2 - 1} = \frac{dx}{x}$$

$$\frac{1}{2} \int \frac{1}{u-1} - \frac{1}{u+1} = \int \frac{1}{x}$$

$$\frac{1}{2} \ln \left| \frac{u-1}{u+1} \right| = \ln|x| + C$$

$$\Rightarrow \sqrt{\left| \frac{u-1}{u+1} \right|} = Ke^x \Rightarrow \boxed{\frac{u-1}{u+1} = Ke^{2x}}$$

$$u-1 = Ke^{2x}(u+1) \Rightarrow u(1 - Ke^{2x}) = Ke^{2x} + 1$$

$$u = \frac{1 - Ke^{2x}}{Ke^{2x} + 1}$$

$$y' + 2y = x^3 e^{-2x}$$

$$y' + 2y = 0 \Rightarrow \frac{y'}{y} = -2 \Rightarrow y_1 = e^{-2x}$$

$$\boxed{y = e^{-2x} \cdot u}$$

$$\Rightarrow y' = u' e^{-2x} - 2u e^{-2x}$$

$$u' e^{-2x} - 2u e^{-2x} + 2u e^{-2x} = x^3 e^{-2x}$$

$$u' e^{-2x} = x^3 e^{-2x}$$

$$u' = x^3 \cdot \text{b/c } e^{-2x} > 0 \text{ for all } x.$$

$$u = \frac{x^4}{4} + C$$

$$y = e^{-2x} \cdot \left(\frac{x^4}{4} + C \right)$$

