Khoi Vo **MATH_046_001_21F**

Assignment 5.1_HOMOGENEOUS_LINEAR_EQUATIONS due 11/13/2021 at 11:59pm PST

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1. (1 point)

It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear** .
Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

? 1. $y'' - y + t^2 = 0$
? 2. $\dfrac{dy}{dt} + ty^2 = 0$
? 3. $y'' - y + y^2 = 0$
? 4. $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + (cos^2(t))y = t^3$

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2. (1 point)

Determine which of the following pairs of functions are linearly independent.

? 1. $f(t) = e^{\lambda t}\cos(\mu t)$, $g(t) = e^{\lambda t}\sin(\mu t)$, $\mu \neq 0$
? 2. $f(x) = x^2$, $g(x) = 4|x|^2$
? 3. $f(\theta) = \cos(3\theta)$, $g(\theta) = 20\cos^3(\theta) - 15\cos(\theta)$
? 4. $f(x) = e^{5x}$, $g(x) = e^{5(x-3)}$

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3. (1 point)

Determine whether the following pairs of functions are linearly independent or not.

? 1. $f(t) = t^2 + 4t$ and $g(t) = t^2 - 4t$
? 2. $f(t) = t$ and $g(t) = |t|$
? 3. $f(x) = e^{4x}$ and $g(x) = e^{4(x-1)}$

2/3. $f = \cos 3\theta$

$$g = 20\cos^3\theta - 15\cos\theta$$
$$= 5\left[\underbrace{4\cos^3\theta - 3\cos\theta}\right]$$
$$= 5\cos(3\theta)$$
$$g = 5f$$

(*) is possible $\Rightarrow$ dependence
$\Rightarrow$ not independence

2/. $W(f,g) = fg' - gf'$ (1)

$= e^{\lambda t}\cos\mu t \cdot \left[\lambda e^{\lambda t}\sin(\mu t) + e^{\lambda t}\mu\cos(\mu t)\right]$

$- e^{\lambda t}\sin(\mu t)\left[\lambda e^{\lambda t}\cos(\mu t) - \mu e^{\lambda t}\sin(\mu t)\right]$

$= \ldots \neq 0$: independent.

2. $W(x^2, \underline{4|x|^2})$ What is $\left[|x|^2\right]' = ?$

dependence means: there is a constant multiply w/ one function will give you the other function (*)

$4|x|^2 = 4x^2$: ~~be~~ because it is a square.

$W(x^2, 4x^2) = x^2 \cdot (8x) - 4x^2 \cdot (2x)$

$= 0$: dependence

$x^2, \overline{4|x|}$: cannot find $[4|x|]'$.

Could you find a number do (*)? NO

→ not dependent → independent.

4/ $f = e^{4x}$, $g = e^{4(x-1)} = \underbrace{e^{4x}}_{f} \cdot e^{-4}$

$g = f \cdot$ number

(*) is possible ⇒ dependence.

1. (1 point) Match the following differential equations with their solutions.
The symbols A, B, C in the solutions stand for arbitrary constants.
You must get all of the answers correct to receive credit.

___1. $\frac{d^2y}{dx^2} + 9y = 0$
___2. $\frac{dy}{dx} = \frac{-2xy}{x^2-3y^2}$
___3. $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 25y = 0$
___4. $\frac{dy}{dx} = 6xy$
___5. $\frac{dy}{dx} + 15x^2y = 15x^2$

A. $y = Ae^{-5x} + Bxe^{-5x}$
B. $y = Ae^{3x^2}$
C. $y = Ce^{-5x^3} + 1$
D. $3yx^2 - 3y^3 = C$
E. $y = A\cos(3x) + B\sin(3x)$

2. (1 point)
Check by differentiation that $y = 4\cos 3t + 6\sin 3t$ is a solution to $y'' + 9y = 0$ by finding the terms in the sum:
$y'' =$ ________________
$9y =$ ________________
So $y'' + 9y =$ _______

3. (1 point)
Solve the following differential equation:

$$y'' - 2y' + 2y = 0$$

and express your answer in the form

$$ce^{\alpha x}\sin(\beta x + \gamma)$$

Answer: $\alpha =$ ___________, $\beta =$ ___________.

4. (1 point)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2} + 15\frac{dy}{dt} + 56y = 0$$

The solution can be written in the form

$$y = C_1e^{r_1t} + C_2e^{r_2t}$$

with

$$r_1 < r_2$$

Using this form, $r_1 =$ _________ and $r_2 =$ _________

5. (1 point) Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2} - 14\frac{dy}{dt} = 0$$

The solution can be written in the form

$$y = C_1e^{r_1t} + C_2e^{r_2t}$$

with

$$r_1 < r_2$$

Using this form, $r_1 =$ _________ and $r_2 =$ _________

6. (1 point)
Find the solution to the boundary value problem:

$$\frac{d^2y}{dt^2} - 13\frac{dy}{dt} + 36y = 0, \quad y(0) = 6, y(1) = 4$$

The solution is $y =$ ____________________________

5/ $\frac{d^2y}{dt^2} - 14\frac{dy}{dt} = 0 : 1 \cdot y'' - 14y' = 0$

$$\boxed{1 \cdot r^2 - 14r^1 = 0} \Leftrightarrow r(r-14) = 0,$$

$$r = 0 \text{ or } r = 14$$

$$y = C_1 e^{0t} + C_2 e^{14t}$$

$$r_1 = 0, \; r_2 = 14.$$

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Assignment 5.2_CONSTANT_COEFF_HOMOGENEOUS_EQUATIONS due 11/13/2021 at 11:59pm PST

1. (1 point) Match the following differential equations with their solutions.
The symbols A, B, C in the solutions stand for arbitrary constants.
You must get all of the answers correct to receive credit.

___1. $\frac{d^2y}{dx^2}+9y=0$
___2. $\frac{dy}{dx}=\frac{-2xy}{x^2-3y^2}$
___3. $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+25y=0$
___4. $\frac{dy}{dx}=6xy$
___5. $\frac{dy}{dx}+15x^2y=15x^2$

A. $y=Ae^{-5x}+Bxe^{-5x}$
B. $y=Ae^{3x^2}$
C. $y=Ce^{-5x^3}+1$
D. $3yx^2-3y^3=C$
E. $y=A\cos(3x)+B\sin(3x)$

2. (1 point)
Check by differentiation that $y=4\cos 3t+6\sin 3t$ is a solution to $y''+9y=0$ by finding the terms in the sum:
$y''=$ ________
$9y=$ ________
So $y''+9y=$ ____

3. (1 point)
Solve the following differential equation:

$$y''-2y'+2y=0$$

and express your answer in the form

$$\boxed{ce^{\alpha x}\sin(\beta x+\gamma)}$$

coeff go with x is β.

Answer: $\alpha=$ ________, $\beta=$ ________.

4. (1 point)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}+15\frac{dy}{dt}+56y=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1=$ ______ and $r_2=$ ______

5. (1 point) Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}-14\frac{dy}{dt}=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1=$ ______ and $r_2=$ ______

6. (1 point)
Find the solution to the boundary value problem:

$$\frac{d^2y}{dt^2}-13\frac{dy}{dt}+36y=0,\quad y(0)=6, y(1)=4$$

The solution is $y=$ ________________

3/ $y''-2y'+2y=0$
$r^2-2r+2=0$
$(r-1)^2=-1$
$r=1\pm 1i$
$e^{1x}(c_1\cos 1x+c_2\sin 1x)$

coeff go with x is 1.
$\boxed{\beta=1}$.

$\alpha=1$

1/ 1. $\frac{d^2y}{dx^2} + 9y = 0$ $\Rightarrow E$

$r^2 + 9 = 0$

$r = 3i$, $r = -3i$

$(r^2 = -9)$

$r = 0 \pm 3i$

$y = \underbrace{e^{0t}}_{1}(c_1 \cos 3t + c_2 \sin 3t)$

$= A\cos 3t + B\sin 3t$

3/ $y'' + 10y' + 25y = 0$

$r^2 + 10r + 25 = 0$ A

$(r+5)^{(2)} = 0$

$r = -5$

$y = Ae^{-5x} + Bxe^{-5x}$

5/ $\frac{dy}{dx} + 15x^2 y = 15x^2$

$y_1 = e^{-\int 15x^2 dx} = e^{-5x^3}$

$y = u \cdot e^{-5x^3} \rightarrow C$

$F = x^2 y - y^3 + C$

4/ $\frac{dy}{y} = 6x\,dx$

$\ln y = 3x^2 + C$

$y = e^{C} \cdot e^{3x^2}$

$= C \cdot e^{3x^2}$ $\Rightarrow B$

2/ $\Rightarrow D$.

$(x^2 - 3y^2)dy = -2xy\,dx$

$\underbrace{2xy}_{M}\,dx + (\underbrace{x^2 - 3y^2}_{N})dy = 0$

$M_y = 2x$, $N_x = 2x$ } exact.

$F: \int M\,dx = x^2 y + f(y)$

$\frac{dF}{dy} = x^2 + f'(y) = N$

$f'(y) = -3y^2$

$\boxed{f(y) = -y^3}$