**Khoi Vo** **MATH_046_001_21F**

**Assignment 5.1_HOMOGENEOUS_LINEAR_EQUATIONS due 11/13/2021 at 11:59pm PST**

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**1.** (1 point)

It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear** .

Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

[?] 1. $y'' - y + t^2 = 0$

[?] 2. $\frac{dy}{dt} + ty^2 = 0$

[?] 3. $y'' - y + y^2 = 0$

[?] 4. $\frac{d^3y}{dt^3} + t\frac{dy}{dt} + (cos^2(t))y = t^3$

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**2.** (1 point)

Determine which of the following pairs of functions are linearly independent.

[?] 1. $f(t) = e^{\lambda t}\cos(\mu t)$ , $g(t) = e^{\lambda t}\sin(\mu t)$ , $\mu \neq 0$

[?] 2. $f(x) = x^2$ , $g(x) = 4|x|^2$

[?] 3. $f(\theta) = \cos(3\theta)$ , $g(\theta) = 20\cos^3(\theta) - 15\cos(\theta)$

[?] 4. $f(x) = e^{5x}$ , $g(x) = e^{5(x-3)}$

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**3.** (1 point)

Determine whether the following pairs of functions are linearly independent or not.

[?] 1. $f(t) = t^2 + 4t$ and $g(t) = t^2 - 4t$

[?] 2. $f(t) = t$ and $g(t) = |t|$

[?] 3. $f(x) = e^{4x}$ and $g(x) = e^{4(x-1)}$

$f = 3t$

$g = |t|$ ?

$3g = 3|t| \overset{?}{=} 3t$

NO

$4/\ f = e^{5x}$ , $g = e^{5(x-3)}$

$= e^{5x} e^{-3}$

$g = f \cdot \boxed{e^{-3}}$ : $\Rightarrow$ not independence

$3/1/\ f = t^2 + 4t$,

$g = t^2 - 4t$

$f = \cos 3\delta$

$g = 2\cos^3\delta - 2\cos\delta$

$= 2\cos\delta\left[\cos^2\delta - 1\right]$

$= 2\cos\delta\ (-\sin^2\delta]$

$3/2.\ f = t$, $g = |t|$

cannot find a scalar

$\Rightarrow$ independence.

2/ (*): not independence = dependence

you can find a number $\boxed{c}$ so that when you multiply one function with $\boxed{c}$, you get the other function. | cannot find $\boxed{c}$ : independence

(**): independence: $W(f,g)=0$

dependence if $W(f,g)\neq 0$.

1/ $W(f,g) = f\cdot g' - f'\cdot g$

$= (e^{\lambda t}\cos \mu t)\cdot\left[e^{\lambda t}\cdot\lambda \sin(\mu t) + e^{\lambda t}\cdot\mu \sin \mu t\right]$

$-(e^{\lambda t}\sin \mu t)\left[e^{\lambda t}\cdot\lambda\cos\mu t - e^{\lambda t}\mu\sin\mu t\right]$

$\neq 0$ : not indep.

2/ $f = x^2$ , $g = 4|x|^2$    $g' = ?$

x

|x|

$f = x^2 = |x|^2$

the scalar is 4: $4f = g$ : NOT indep.

$4|x|^2 = g$

3/ $f = \cos 3\theta$ , $g = 20\cos^3\theta - 15\cos\theta$

$= 5\left[4\cos^3\theta - 3\cos\theta\right]$

$g = 5\cdot\cos 3\theta = 5f$

you can find constant $c=5 \Rightarrow$ NOT independence

**1.** (1 point) Match the following differential equations with their solutions.
The symbols $A$, $B$, $C$ in the solutions stand for arbitrary constants.
You must get all of the answers correct to receive credit.

___1. $\frac{d^2y}{dx^2}+9y=0$
___2. $\frac{dy}{dx}=\frac{-2xy}{x^2-3y^2}$
___3. $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+25y=0$
___4. $\frac{dy}{dx}=6xy$
___5. $\frac{dy}{dx}+15x^2y=15x^2$

A. $y=Ae^{-5x}+Bxe^{-5x}$
B. $y=Ae^{3x^2}$
C. $y=Ce^{-5x^3}+1$
D. $3yx^2-3y^3=C$
E. $y=A\cos(3x)+B\sin(3x)$

**2.** (1 point)
Check by differentiation that $y=4\cos 3t+6\sin 3t$ is a solution to $y''+9y=0$ by finding the terms in the sum:
$y''=$ ________
$9y=$ ________
So $y''+9y=$ ____

**3.** (1 point)
Solve the following differential equation:

$$y''-2y'+2y=0$$

and express your answer in the form

$$ce^{\alpha x}\sin(\beta x+\gamma)$$

Answer: $\alpha=$ ________, $\beta=$ ________.

**4.** (1 point)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}+15\frac{dy}{dt}+56y=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1=$ ______ and $r_2=$ ______

**5.** (1 point) Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}-14\frac{dy}{dt}=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1=$ ______ and $r_2=$ ______

**6.** (1 point)
Find the solution to the boundary value problem:

$$\frac{d^2y}{dt^2}-13\frac{dy}{dt}+36y=0,\quad y(0)=6, y(1)=4$$

The solution is $y=$ ________________

$y''-2y'+2y=0$

$t^2-2t+2=0$

$t=1\pm 1i$ ($\alpha$, $\to\beta$)

$y=e^{1x}[\cos(1x)+\sin 1x]$

$\alpha=1,\ \beta=1.$

$\dfrac{2\pm\sqrt{-4}}{2}$ ← $4-4\cdot 2$

$6=c_1e^0+c_2e^0 : 6=c_1+c_2$

$4=c_1e^4+c_2e^9$

$4=(6-c_2)e^4+c_2e^9$

$=6e^4+(e^9-e^4)c_2$

$c_2=\dfrac{4-6e^4}{e^9-e^4}=0.$

4/ $1y'' + 15y' + 56y = 0$

Characteristic function:

$r^2 + 15r + 56 = 0$

$(r+7)(r+8)$

$r = -7; \; r = -8$

$y = c_1 e^{-8t} + c_2 e^{-7t}$

$y = e^{\alpha x}(\cos \beta x + \sin \beta x)$

5/ $y'' - 14y' = 0$

$r^2 - 14r = 0 \quad r = 0, \quad 14$

$y = c_1 e^{0t} + c_2 e^{14t}$

(*) double root, 2

$y = c_1 e^{2x} + c_2 \cdot \boxed{x} \cdot e^{2x}$

$(t-2)^2 = 0$

$(t-2)(t-2)$

6/ $y'' - 13y' + 36 = 0$

$r^2 - 13r + 36$

$(r-4)(r-9) \quad r = 4, r = 9$

$y = c_1 e^{4t} + c_2 e^{9t}$

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**Assignment 5.2_CONSTANT_COEFF_HOMOGENEOUS_EQUATIONS due 11/13/2021 at 11:59pm PST**

**1.** (1 point) Match the following differential equations with their solutions.
The symbols $A$, $B$, $C$ in the solutions stand for arbitrary constants.
You must get all of the answers correct to receive credit.

___1. $\frac{d^2y}{dx^2} + 9y = 0$
___2. $\frac{dy}{dx} = \frac{-2xy}{x^2-3y^2}$
___3. $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 25y = 0$
___4. $\frac{dy}{dx} = 6xy$
___5. $\frac{dy}{dx} + 15x^2y = 15x^2$

A. $y = Ae^{-5x} + Bxe^{-5x}$
B. $y = Ae^{3x^2}$
C. $y = Ce^{-5x^3} + 1$
D. $3yx^2 - 3y^3 = C$
E. $y = A\cos(3x) + B\sin(3x)$

**2.** (1 point)
Check by differentiation that $y = 4\cos 3t + 6\sin 3t$ is a solution to $y'' + 9y = 0$ by finding the terms in the sum:
$y'' =$ ________________
$9y =$ ________________
So $y'' + 9y =$ _______

**3.** (1 point)
Solve the following differential equation:

$$y'' - 2y' + 2y = 0$$

and express your answer in the form

$$ce^{\alpha x}\sin(\beta x + \gamma)$$

Answer: $\alpha =$ ___________, $\beta =$ ___________.

**4.** (1 point)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2} + 15\frac{dy}{dt} + 56y = 0$$

The solution can be written in the form

$$y = C_1e^{r_1t} + C_2e^{r_2t}$$

with

$$r_1 < r_2$$

Using this form, $r_1 =$ _________ and $r_2 =$ _________

**5.** (1 point) Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2} - 14\frac{dy}{dt} = 0$$

The solution can be written in the form

$$y = C_1e^{r_1t} + C_2e^{r_2t}$$

with

$$r_1 < r_2$$

Using this form, $r_1 =$ _________ and $r_2 =$ _________

**6.** (1 point)
Find the solution to the boundary value problem:

$$\frac{d^2y}{dt^2} - 13\frac{dy}{dt} + 36y = 0, \quad y(0) = 6, y(1) = 4$$

The solution is $y =$___________________________