Math 31 - Fall 2021 - Discussion 15 - November 18

Topic: Sections 4.6 and 3.1.

1. Let
$$A = \begin{pmatrix} 1 & -9 & 10 & -11 \\ -1 & 6 & -5 & 5 \\ 6 & -30 & 20 & -18 \end{pmatrix}$$
Find rank(A), dimension of Nul(A), and find bases for Col(A), Row(A) and Nul(A).

2. If a 4×7 matrix has rank 2, then find the dimension of Nul(A), the dimension of Row(A), and rank(A^T).

3. a. Suppose A is a 5×8 matrix and that has 5 pivot columns. Is $\text{Col}(A) = \mathbb{R}^5$? Is $\text{Nul}(A) = \mathbb{R}^3$?
   b. If A is a 5×2 matrix, what is the largest possible rank of A? And if A is a 2×5 matrix, what is the largest possible rank of A?
   c. If A is a 7×9 matrix, what is the largest possible dimension of row space of A? And if A is a 9×7 matrix, what is the largest possible dimension of row space of A?

4. Compute the determinant of
$$A = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 3 & 1 \\ 0 & 4 & 1 \end{pmatrix}$$

$$[A \mid 0] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & -5 & 7 & 0 \\ 0 & -3 & 5 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_3 free, x_4 free

$-3x_2 = -5x_3 + 6x_4$

$x_2 = \frac{5}{3}x_3 - 2x_4$

$x_1 - 5x_3 + 7x_4 = 0 \;:\; x_1 = 5x_3 - 7x_4$

$$\begin{bmatrix} 5x_3 - 7x_4 \\ \frac{5}{3}x_3 - 2x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ \frac{5}{3} \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

basis of Nul(A) $= \left\{ \begin{bmatrix} 5 \\ \frac{5}{3} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

// * rank (A): dimension of Col (A)

* Nul (A) basis: $Ax = 0$

$[A \mid 0]$

* basis of Row (A): row space of A

1/ row reduced echelon A into B.

2/ The non-zero rows of B is the basis of Row(A).

$$\begin{bmatrix} 1 & -9 & 10 & -11 \\ -1 & 6 & -5 & 5 \\ 6 & -30 & 20 & -18 \end{bmatrix} \xrightarrow[R_2+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & -9 & 10 & -11 \\ 0 & -3 & 5 & -6 \\ 0 & 6 & -10 & 12 \end{bmatrix} \times(-2)$$

$$\to \begin{bmatrix} 1 & -9 & 10 & -11 \\ 0 & -3 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -5 & 7 \\ 0 & -3 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

non-zero rows of "B"

$\Rightarrow$ Basis for Row A $= \{(1\ 0\ -5\ 7), (0\ -3\ 5\ -6)\}$

(*) Column space basis = basis of Col (A)

= " pivots column of A ": first column, second column

$$= \left\{ \begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} -9 \\ 6 \\ -30 \end{bmatrix} \right\}$$

rank A = 2.

base for column(A)

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21 $4 \times \boxed{7}$ matrix

rank A = 2

dim Nul (A) = ? 5 ✓

dim Row (A) = ? 2 ✓

rank (A^T) = dim of col (A^T) = dim of Row (A)

= 2.

Rank and Nulity Theorem:

Rank A + dim Nul (A) = number of column in A

2 + 5 = 7

* Rank A = dim of Row (A) = 2 = numbers of pivot columns.

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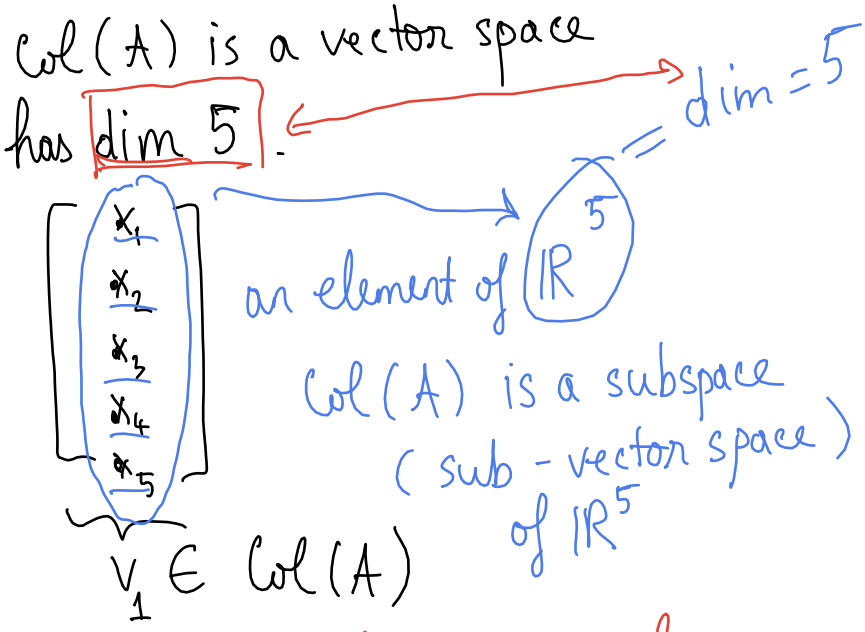
3/a : 5 by 8 , 5 pivot columns .

dim of Col(A) = # of pivot columns
= 5

Col(A) is a vector space
has dim 5 .



subspace same dim so equal :

$Col(A) = \mathbb{R}^5$.

dim Nul(A) = # of free variables
= 3

s_1

5 by 8

8 by 1

$\begin{bmatrix} 8 \\ \text{entries} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

element $\mathbb{R}^8$

Nul A $\neq \mathbb{R}^3$

$\begin{bmatrix} 8 \\ \text{entries} \end{bmatrix}, \begin{bmatrix} \quad \end{bmatrix}, \begin{bmatrix} \quad \end{bmatrix}$

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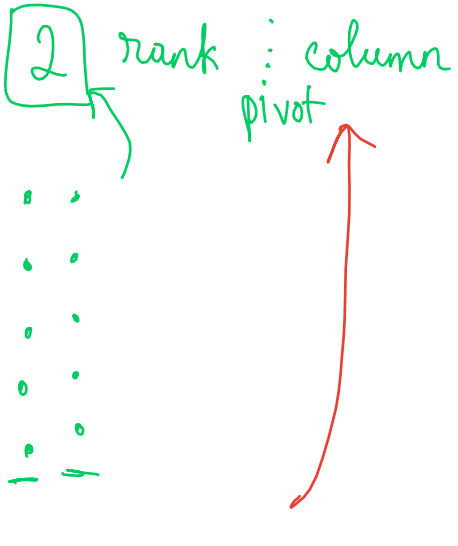

2 by 5

dim of Row = rank

c/7 : 7 pivot columns

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"sign"

$+3 \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} - 0 \begin{vmatrix} \quad \end{vmatrix}$

$+1 \begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix}$

$= 3 \cdot (-1) - 0 + 1 \cdot 8 = 5$