1. (1 point)
Let $y(t)$ be the solution to $\dot{y} = 8te^{-y}$ satisfying $y(0) = 3$.
(a) Use Euler's Method with time step $h = 0.2$ to approximate $y(0.2), y(0.4), ..., y(1.0)$.

k	t_k	y_k
0	0	3
1	0.2	___
2	0.4	___
3	0.6	___
4	0.8	___
5	1.0	___

(b) Use separation of variables to find $y(t)$ exactly.
$y(t)$ = __________
(c) Compute the error in the approximations to $y(0.2)$, $y(0.6)$, and $y(1)$.
$|y(0.2) - y_1| =$ __________
$|y(0.6) - y_3| =$ __________
$|y(1) - y_5| =$ __________
Answer(s) submitted:

-
-
-
-
-
-
-
-
-

(incorrect)

3. (1 point) Use Euler's method with the given step size to estimate $y(1.4)$ where $y(x)$ is the solution of the initial-value problem

$$y' = x - xy, \quad y(1) = 1.$$

1. Estimate $y(1.4)$ with a step size $h = 0.2$.
Answer: $y(1.4) \approx$ __________
2. Estimate $y(1.4)$ with a step size $h = 0.1$.
Answer: $y(1.4) \approx$ __________

Answer(s) submitted:

-
-

(incorrect)

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1. (1 point)

A continuous annuity with withdrawal rate N = \$1,400/year and interest rate r = 5

(a) When will the annuity run out of funds if $P_0 = \$23{,}000$?

The annuity runs out after approximately ________ years.

Answer to the nearest whole year.

(b) Which initial deposit P_0 yields a constant balance? $P_0 =$ \$ ________

Answer(s) submitted:

-
-

(incorrect)

The instantaneous rate of change of the value of a certain investment (P) is proportional to its value. That is to say $\frac{dP}{dt} = rP$.

If $r = 7$ and $P(0) = 3500$:

$P(t) =$ ________________.

Answer(s) submitted:

-

(incorrect)

$$P(t) = \frac{-N}{r} + Ce^{rt} = \boxed{\frac{-1400}{0.05}} + Ce^{0.05t}$$

Withraw

28,000

$$23{,}000 = \frac{-1400}{0.05} + Ce^{0} \Rightarrow C = 5{,}000$$

$$\boxed{P(t) = -28{,}000 + 5{,}000e^{0.05t} = 0}$$

$$5{,}000e^{0.05t} = 28{,}000$$

$$e^{0.05t} = \frac{28}{5}$$

$$0.05t = \ln\left(\frac{28}{5}\right)$$

$$t = \frac{}{0.05} = 34.4 \approx 34$$

b) $0 = -P_0 + 5{,}000e^{0.05t}$

$P_0 = 28{,}000$

$-(28{,}000) = -(28{,}000) + 5000e^{0}$

$\Rightarrow$

Khoi Vo **MATH_046_001_21F**

Assignment 4.2_Cooling_and_Mixing due 11/06/2021 at 11:59pm PDT

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1. (1 point) A tank contains 100 kg of salt and 2000 L of water. Pure water enters a tank at the rate 6 L/min. The solution is mixed and drains from the tank at the rate 3 L/min.

(a) What is the amount of salt in the tank initially?
amount = ________ (kg)

(b) Find the amount of salt in the tank after 5 hours.
amount = ________ (kg)

(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)
concentration = ________ (kg/L)

Answer(s) submitted:

-
-
-

(incorrect)

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2. (1 point) A tank contains 100 kg of salt and 2000 L of water. A solution of a concentration 0.025 kg of salt per liter enters a tank at the rate 7 L/min. The solution is mixed and drains from the tank at the same rate.

$r =$

a.) What is the concentration of our solution in the tank initially?

concentration = $\frac{200}{2000}$ (kg/L)

b.) Find the amount of salt in the tank after 4.5 hours.

amount = ________ (kg)

c.) Find the concentration of salt in the solution in the tank as time approaches infinity.

concentration = ________ (kg/L)

Answer(s) submitted:

0.025

-
-
-

(incorrect)

$100e^{-7*4.5*60/2000}$: in

$\boxed{0.025*2000}\,e^{-7*4.5*60/2000}$: out

$$\frac{1}{t^2-1} = \frac{1}{2}\frac{1}{t-1} - \frac{1}{2}\frac{1}{t+1}$$

$$x^2 y' = y^2 + xy - x^2, \quad y(1) = 2$$

$\underline{\underline{x^2 = 0:}}$ $0 = y^2 + 0 - 0 \Rightarrow y = 0$.

$x^2 \neq 0:$ $y' = \dfrac{y^2}{x^2} + \dfrac{y}{x} - 1.$

$t = \dfrac{y}{x}:$ $y' = t^2 + t - 1$

$y = tx$

$y' = t'x + t.$

$$t'x + \cancel{t} = t^2 - 1 \cancel{+t}$$

$$t'x = t^2 - 1$$

$$\frac{dt}{dx} x = t^2 - 1$$

$$\frac{dt}{t^2-1} = \frac{dx}{x}$$

$$\int \frac{dt}{t^2-1} = \frac{1}{2} \int \left(\frac{1}{t-1} - \frac{1}{t+1} \right) dt = \ln x + C$$

$$\frac{1}{2} \ln\left(\frac{t-1}{t+1}\right) = \ln x + C$$

$$\frac{1}{t^2-1} = \frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$$

Find A and B: $1 = A(t+1) + B(t-1)$

Set $t = -1$: $1 = A \cdot 0 + B(-1-1)$

$1 = 0 - 2B \Rightarrow B = \dfrac{-1}{2}$

Set $t = 1$: $1 = A \cdot 2 + B \cdot 0 = A \cdot 2 \Rightarrow A = \dfrac{1}{2}$