

145B, Week 3

* $\mathbb{R}_\ell$ = lower limit topo on $\mathbb{R}$

Question 1: Can you list the "open" sets in $\mathbb{R}_\ell$?

Any interval $[a, b)$ with $a < b$, a, b are real numbers
because the basis $\mathcal{B}_\ell = \{ [a, b) \}$ generate $\mathbb{R}_\ell$.

Question 2: Is (a, b) open in $\mathbb{R}_\ell$?

Ans: yes. Because $(a, b) = \bigcup_{x \in (a, b)} [x, b)$

arbitrary union of open $[x, b)$
is open.

* (a): connected components of $\mathbb{R}_\ell$?

Defn: (connected components of X):

(Actually, more of a "Theorem" 2.9 in the note)

They are the disjoint subspaces of X whose union is X .
connected !!

Following hints, we claim that connected components of $\mathbb{R}_\ell$ are the singleton $\{x\} : x \in X$.

So we need to find "connected" components of $\mathbb{R}_\ell$.

Let C be a connected component of $\mathbb{R}_\ell$.

Assume $\emptyset \neq C$ and says C has at least 2 elements.
says $a, b \in C$. Of course we can assume $a < b$.

Question 3: Is $(-\infty, x)$ open in $\mathbb{R}_\ell$?

Ans: Yes. Because $(-\infty, x) = \bigcup_{n \in \mathbb{N}} [x - \frac{1}{n}, x)$

Question 4: Is $[x, \infty)$ open in $\mathbb{R}_\ell$

Ans: Yes. Because $[x, \infty) = \bigcup_{n \in \mathbb{N}} [x, x + \frac{1}{n})$

Then $(-\infty, b)$ and $[b, \infty)$ are open in $\mathbb{R}_\ell$
contain a contain b

So: $K = (-\infty, b) \cap C$ is not empty (contains a)
and open in $\mathbb{R}_\ell$

Similarly $H = [b, \infty) \cap C$ is not empty (contains b)
and open in $\mathbb{R}_\ell$.

Now look at C as a space $\nearrow$

$C = K \cup H$, so K and H is a separation of C

$\Rightarrow C$ not connected. A contradiction!!!

So C can have at most 1 element.

Thus C is the empty set or the singletons !!!

b/ Determine the path components of $\mathbb{R}_\ell$

Theorem 2.9 (note) : are the path connected, disjoint whose union is $\mathbb{R}$.

Let C be the path connected component.

By Theorem 2.8, a path connected is connected.

So C must also be connected.

So C is either empty set or singletons !!!

c/ f continuous $\mathbb{R}_{\text{std}} \rightarrow \mathbb{R}_\ell$.

Proposition 2.2 : $\mathbb{R}_{\text{std}}$ is connected

Theorem 2.6 : f is continuous. Then f send connected space to connected space.

So $f(\mathbb{R}_{\text{std}})$ must be connected in $\mathbb{R}_\ell$.

But the connected component in $\mathbb{R}_\ell$ are singleton, $\{c\}$

So $f(\mathbb{R}_{\text{std}}) = \{c\}$ for some $c \in \mathbb{R}$

Thus the continuous maps are the constant maps.

[illegible]