Math 135A a note on Scale Partial Pivoting

Monday, November 28, 2022 9:35 AM

+ The system in the exercise is:

$$\left[\begin{array}{cccc:c} -1 & 1 & 0 & -3 & 4 \\ 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right]$$

(*) The scale vector $\underline{S} = [\overset{S_1}{3} \;\; \overset{S_2}{3} \;\; \overset{S_3}{1} \;\; \overset{S_4}{3}]$

How to find S : maximum value of each row in <u>absolute</u> value

<u>eg</u>: 1^{st} row: $-1 \quad 1 \quad 0 \quad -3$ $\quad -3 \to$ abs value is $\underline{3}$

2^{nd} row: $1 \quad 0 \quad 3 \quad 1$: max abs is $\underline{3}$

3^{rd} row: $0 \quad 1 \quad -1 \quad -1$: max abs is $\underline{1}$

4^{th} row: $3 \quad 0 \quad 1 \quad 2$: max abs is $\underline{3}$

<u>Important reminder</u>: this S is "fixed".

Next we start with index vector $\underline{\ell} = [\underset{\ell_1}{1} \;\; \underset{\ell_2}{2} \;\; \underset{\ell_3}{3} \;\; \underset{\ell_4}{4}]$

Right now we have $\ell_1 = 1$, $\ell_2 = 2$, $\ell_3 = 3$, $\ell_4 = 4$

Later we will see these ℓ_i values change!!!
(because we switch the row)

+ <u>scale pivot</u>: take entries in $\boxed{1^{st} \text{ column}}$ divided by the S

first row of column 1 $\qquad\qquad\qquad\qquad$ 4^{th} row of column 1

$$\frac{-1}{3} \qquad \frac{1}{3} \qquad \frac{0}{1} \qquad \frac{3}{3}$$

$S_1 \qquad S_2 \qquad S_3 \qquad S_4$ $\quad$ always fixed the "s"

We want $\boxed{\text{BIGGEST}}$: so it is $\frac{3}{3} \to$ meaning 4^{th} row

So we will "switch 4^{th} row and $\boxed{1^{st} \text{ row}}$

why 1^{st} ?

in the <u>index vector</u> $\qquad$ because we are taking numbers from $\boxed{1^{st} \text{ column}}$

$\ell = [4 \;\; 2 \;\; 3 \;\; 1]$

Also, we will fix the 4^{th} row and try to turn the other rows into $\underline{\underline{0}}$ (only in the first column)

by $\boxed{\text{multipliers}}$

try to get 0

try to get 0

try to get 0

$$\left[\begin{array}{cccc|c} \_ & \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \\ 3 & 0 & 1 & 2 & 1 \end{array}\right]$$

how do we get the multipliers: (see the red and blue)

$$\left[\begin{array}{cccc|c} -1 & 1 & 0 & -3 & 4 \\ 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow[R_2-\left(\frac{1}{3}\right)R_4]{R_1-\left(-\frac{1}{3}\right)R_4} \left[\begin{array}{cccc|c} 0 & 1 & 1/3 & -7/3 & 13/3 \\ 0 & 0 & 8/3 & 1/3 & -1/3 \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right]$$

already 0 (do nothing)

Recall we get the new $l = [\,4 \quad 3 \quad 2 \quad 1\,]$

we fix the $S = [\,\underset{s_1}{3} \quad \underset{s_2}{3} \quad \underset{s_3}{1} \quad \underset{s_4}{3}\,]$

Now we use <u>second</u> column $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

and <u>divide</u> the S, but do by the order in the l (pay attention to the colors I use)

$l = [\,\cancel{4} \quad 2 \quad 3 \quad 1\,]$

used already

second entries $\dfrac{0}{3}$ ← s_2

third entry $\dfrac{1}{1}$ ← s_3

first entries in column $\dfrac{1}{3}$ ← s_1

which one is BIGGEST? $\underset{1}{1}$ third entries → third row

$\boxed{\text{keep third row}}$

also get <u>new</u> index vector: $l = [\,4 \quad 3 \quad 2 \quad 1\,]$

$l_3 \leftrightarrow l_2$

When we keep third row:

$$\left[\begin{array}{cccc|c} 0 & 1 & - & - & - \\ 0 & 0 & - & - & - \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_1 - \left(\frac{1}{1}\right) R_3} \left[\begin{array}{cccc|c} 0 & 0 & \frac{4}{3} & -\frac{4}{3} & \frac{4}{3} \\ 0 & 0 & 8/3 & 1/3 & -1/3 \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right]$$

turn to 0

turn to 0 → already 0

* Now for third column: $\begin{bmatrix} 4/3 \\ 8/3 \\ -1 \\ 1 \end{bmatrix}$

Recall $S = \begin{bmatrix} \underset{s_1}{3} & \underset{s_2}{3} & \underset{s_3}{1} & \underset{s_4}{3} \end{bmatrix}$

and our index vector is $l = [\,4 \;\; 3 \;\; 2 \;\; 1\,]$

scale pivot

$$\frac{\frac{8}{3}}{3} = \frac{8}{9} \qquad \frac{\frac{4}{3}}{3} = \frac{4}{9}$$

s_2 $\qquad$ s_1

"blue" color → second row is pivot → keep second row

and index is $l = [\,4 \;\; 3 \;\; 2 \;\; 1\,]$

third column and second row

need to interchange this in l

but we already did that earlier

⇒ <u>no</u> change

turn this into 0

$$\left[\begin{array}{cccc|c} 0 & 0 & \frac{4}{3} & -\frac{4}{3} & \frac{4}{3} \\ 0 & 0 & \frac{8}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_1 - \overset{1/2}{\frac{\frac{4}{3}}{\frac{8}{3}}} R_2} \left[\begin{array}{cccc|c} 0 & 0 & 0 & -\frac{3}{2} & \frac{3}{2} \\ 0 & 0 & \frac{8}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -1 & -1 & 3 \\ 3 & 0 & 1 & 2 & 1 \end{array}\right]$$

Now you can solve this w/ backward substitution ⇒ $x_4 = \ldots$