Some practice questions for Exam 2 - Math 115 Business Calculus
California State University, Long Beach
Fall 2017

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October 22, 2017

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Contents

Chapter 1

Beginning Remark

Dear students,

Please keep in mind as you are reading/working on this paper that this is not a study guide, and the topics provided in this paper is **NOT** a complete list of topics that will appear on the exam.

In fact, this is just a list of problems that I think you should know, and as if I were studying for this test, I should know these topics. I also try to include suggestion on how to approach these problems. I took my personal time to compile this list in hoping that it could help you study for your exams. I know that you all have a background from economics and business related, so mathematics might be a foreign land to some of you. But, as I always believe, hard work and diligence always bring sweet fruits in the end. So please use this paper as a good supportive resource to you.

Please let me know if you have any suggestion on this work. Again, this work is just a work in progress, and it needed a lot of suggestion for the better learning experiences.

Study hard and good luck on your exams.
Best regards,
Khoi Vo.

Chapter 2

Review Materials

2.1 Section 2.2: Exponential Functions and Exponential Models

A typical exponential model has the equation $f(x) = Ab^x$. We called b the base, A and b are both real numbers. Keep in mind that x is the **input** and $f(x)$ is the **output**. Pay closed attention if the problems have words like **grow exponentially**, in particular, the word **exponentially** suggested somewhere an **exponential model**. If you were asked to find an exponential model, or find an exponential equation, your job is to find A and b.
Also, you were introduced the **number** e. This e is a number !!!.

Exercise 1. *Determine which function is linear, which function is exponential, and which function is neither linear nor exponential. If the function is linear, find the slope and write out its equation. If the function is exponential, find its base and write its equation.*

1. *For function f and g:*

x	-2	-1	0	1	2
$f(x)$	100	200	300	400	500
$g(x)$	100	20	4	0.8	0.16

2. *For function h and k*

x	-2	-1	0	1	2
$h(x)$	0.8	0.2	0.1	0.05	0.025
$k(x)$	80	40	20	10	2

Answer: f is linear with slope $m = 100$ and $f(x) = 100x + 300$. g is exponential with base $b = 0.2$, so $g(x) = 4(0.2)^x.h$ and k are neither linear nor exponential.

Exercise 2. *World population was estimated at* 2.56 *billion people in* 1950 *and* 6.40 *billion people in* 2004.

1. *Use these data to give an exponential growth model showing the world population P as a function of time t in years since 1950. Round coefficients to five decimal places.*

2. *Assuming the exponential growth model from part (1), estimate the world population in 2020.*

Answer: 1. $P(t) = 2.56(1.02317)^t$
2. $A(70) = 12.72289$ billions people

Exercise 3. *South African Breweries (SAB) reported fixed assets of* 360 *million dollars in* 1997 *and* 480 *million dollars in* 2000 . *Use this information to find*

1. *a linear model and an exponential model for SABs fixed assets as a function of time t since* 1997. *(Round all coefficients to four significant digits.)*

2. *Which, if either, of these models would you judge to be applicable to the data shown below?*

year	1997	1998	1999	2000	2001
SAB Profit (million dollars)	360	380	320	480	360

Note 1: Remember to change from the year to the time t. $t = 0$ is year 1997.
Answer: For 1. $L(t) = 40t + 360$ is linear model, and $P(t) = 360(1.1006)^t$ is the exponential model .
For 2, Neither model is applicable.

2.2 Section 4.2: Marginal Analysis: Marginal Cost, Marginal Profits

Definition 1. The Marginal Cost is the cost of producing 1 additional unit of input.
The Marginal Cost is calculated as derivatives of the cost function.
The Marginal Cost is used to estimated/approximated the cost of producing 1 additional unit.

Exercise 4. Suppose that the cost in hundred dollars to manufacture surface book is given by $C(x) = 150,000 + 20x - 0.0001x^2$ where x is the number of surface books manufactured.

1. what is the rate of increasing of the cost when produce $50,000$ surface books?

2. Find the marginal cost function and use it to estimate the cost when producing $50,001$**st** surface books.

2.3 Section 4.3: The Product and Quotient Rule

Definition 2. Product Rule: If $f(x)$ and $g(x)$ are differentiable functions of x, then so is their product $f(x)g(x)$, and

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

To remember **product rule in words**:
The derivative of a product is (the derivative of the first) **times** (the second), plus (the first) **times** (the derivative of the second).

Definition 3. Quotient Rule: If $f(x)$ and $g(x)$ are differentiable functions of x, then so is their quotient $f(x)/g(x)$ (provided $g(x) \neq 0$), and

$$\frac{d}{dc}(f(x)/g(x)) = \frac{f'g - fg'}{g^2}$$

Quotient Rule in Words: The derivative of a quotient is (the derivative of the top) **times** (the bottom), **minus** (the top) **times** (the derivative of

the bottom), all over the **bottom squared**.
Notes 1. **Do not** try to remember the rules by the symbols we have used, but remember them in words. (The slogans are easy to remember, even if the terms are not precise.)
2. One more time: The derivative of a product is NOT the product of the derivatives, and the derivative of a quotient is NOT the quotient of the derivatives.

Exercise 5. Find the derivatives of

1. $\frac{1}{\sqrt{x}}$
2. $\frac{2x}{x+2}$
3. $x^2(5x^3 - 4x^2 - 2x)$

Answers:

1.
$$\frac{-1}{2x^{3/2}}$$
2.
$$\frac{4}{(x+2)^2}$$
3.
$$2x(5x^3 - 4x^2 - 2x) + x^2(15x^2 - 12x - 2) = 25x^4 - 20x^3 - 6x^2$$

Exercise 6. Saudi Oil Revenues: The spot price of crude oil during the period 20002005 can be approximated by

$$P(t) = 5t + 25 \textbf{ dollars per barrel, } (0 \leq t \leq 5)$$

in year t, where $t = 0$ represents 2000.

Saudi Arabias crude oil production over the same period can be approximated by

$$Q(t) = 0.082t^2 - 0.22t + 8.2 \textbf{ million barrels per day, } (0 \leq t \leq 5).$$

Use these models to estimate Saudi Arabias daily oil revenue and also its rate of change in 2001. (Round your answers to the nearest 1 million.)
Answer: 1. $R(t) = Q(t)P(t) = (5t + 25)(0,082t^2 - 0.22t + 8.2)$
2.$R'(1) = 96.17$ millions dollars per year.

2.4 Section 4.4: The Chain Rule

Definition 4. Chain Rule If f is a differentiable function of u and u is a differentiable function of x, then the composite $f(u)$ is a differentiable function of x, and

$$\frac{df}{dx} = f'\frac{du}{dx}$$

Chain Rule In words The derivative of f(quantity) is the derivative of f, evaluated at that quantity, **times** the derivative of the quantity.

Note 1: It is very important to keep in mind the input and the output.

Exercise 7. Compute the following derivatives:

1. $\frac{d}{dx}[(2x^2 + x)^3]$
2. $\frac{d}{dx}[(x^3 + x)^{100}]$
3. $\sqrt{3x + 1}$

Answer:

1. $$\frac{d}{dx}[(2x^2 + x)^3] = 3(2x^2 + x)^2(4x + 1)$$
2. $$\frac{d}{dx}[(x^3 + x)^{100}] = 100(x^3 + x)^{99}(3x^2 + 1)$$
3. $$\frac{d}{dx}[\sqrt{3x + 1}] = \frac{3}{2(3x + 1)^{1/2}}$$

Now you also need to know how to set up the Chain Rule in Word Problems. Figure out the input and the output is extremely important!! Set up the Chain Rule is also very important. Take a look at an old example that we did in activity, but this time the functions will be given to you:

Example 1. Marginal Product: Paramount Electronics has an annual profit given by $P(q) = 100,000 + 5000q - 0.25q^2$, where q is the number of laptop computers it sells each year. The number of laptop computers it can make and sell each year depends on the number n of electrical engineers Paramount employs, according to the equation $q(n) = 30n + 0.01n^2$

1. What is the profit of Paramount when they employ 40 electrical engineers?

2. How fast the Profit is changing when they increase the number of electrical engineers they start to employ 50 engineers?

Suggested Ideas: 1. First identify the input and the output in $P(q)$ and in $q(n)$. Remember that the problem did not ask for $P(40)$, because $P(40)$ means.... what is the profit when they make and sell 40 laptops [the input of P is q] !!. However, the problem ask for... $P(q(40))$. Take your time review the note and understand clearly this part here. After that, we calculate $q(40) = 30(40) + 0.01(40)^2 = 1216$. Then $P(q(40)) = P(1216) = 100,000 + 5000(1216) - 0.25(1216)^2 = 5810336$ **dollars**.

2. This is the derivatives of P with respect to n, you can write it up as $\frac{dP}{dn}$. Since there is no direct relationship between input n and output P, we recognize that we must use chain rule. The chain rule said:

$$\frac{d}{dn}P = \frac{dP}{dn}(\frac{dq}{dn})$$

You could set up the chain rule like this, and keep in mind that $\frac{dP}{dq}$ is derivative of P with respect to q (that is, the input is q and the output is P); and that $\frac{dq}{dn}$ is the derivative of q with respect to n (that is, the input is n and the output is q).
Then we calculate $\frac{dP}{dq} = P'(q)$ with $P(q) = 100,000 + 5000q - 0.25q^2$, so $P'(q) = 5000 - 0.5q$. We also calculate $\frac{dq}{dn} = q'(n)$ with $q(n) = 30n + 0.01n^2$, so $q'(n) = 30 + 0.02n$.
Now the problem said they started to employ 50 engineers, so $n = 50$. So $q(50) = 100,000 + 5000(50) - 0.25(50)^2 = 350625$, this is the **input** for the $P'(q)$. So $P'(q) = P'(350625) = 5000 - 0.5(350625) = -170312.5$. Take your time to understand this part where we need to find the input q for the $P'(q)$.
For the other derivatives, take $n = 50$ to plug in $q'(n)$, so $q'(50) = 30 + 0.02(50) = 31$
Now putting this all together, that is, multiply the two derivatives:

$$(-170312.5)(31) = -5279687.5 \textbf{ dollars per engineer.}$$

This means they are losing profit (loss) when they increase engineer if they already started hiring 50 engineers.

2.5 Section 4.5: The Derivative of Exponential Function

Definition 5. If b is any number, and u is a function of x, then

$$\frac{d}{dx}[b^u] = b^u(\ln b)(\frac{du}{dx})$$

IN WORDS: The derivative of b raised to a quantity u is: b raised to that quantity u, times $\ln b$, times the derivative of that quantity u.

Trick to remember: Keep whatever that is, then multiply with ln of the number b, then take derivatives of the power.

Note: if b is the number e, then $\ln b = \ln e = 1$.

Example 2. Calculate derivatives of

1. 5^{x^2+6x+4}
2. e^{5x}
3. $4e^{6x}$
4. $e^{2x}4^{2x}$

Suggested Hints:

1. $5^{x^2+6x+4}(\ln 5)(2x+6)$, where we keep the original 5^{x^2+6x+4}, we multiply with $\ln 5$, and we also multiply with the derivatives of the power, which is the derivative of (x^2+6x+4), so $(x^2+6x+4)' = 2x+6$

2. $e^{5x}(\ln e)(5) = 5e^{5x}$, since $\ln e = 1$. And 5 is the derivative of the power.

3. Leave 4 for alone, it is just a number multiply with the the exponential in calculating derivatives[Just like when you calculate derivative of $5x^2$, you leave 5 alone, get the derivative of x^2 is $2x$, so all together take 5 multiply with $2x$]. So Let's calculate derivative of e^{6x}. The derivative of $e^{6x} = e^{6x}(\ln e)(6) = 6e^{6x}$. Thus the derivative of $4e^{6x}$ is 4 times with $6e^{6x}$, which is $24e^{6x}$.

4. Here we need to use the **product rule**. The product rule applied here gives:

$$[e^{2x}]'(4^{2x}) + (e^{2x})[4^{2x}]'$$

Now then you need to calculate $[e^{2x}]'$ and $[4^{2x}]'$. So $[e^{2x}]' = e^{2x}(\ln e)(2) = 2e^{2x}$; and $[4^{2x}]' = 4^{2x}(\ln 4)(2)$. Put this back into the product rule above, gives the derivative is

$$(2e^{2x})(4^{2x}) + (e^{2x})(2\ln 4)(4^{2x})$$

2.6 Section 5.6: Elasticity

The formula for Price Elasticity is:

$$E(p) = \frac{-pq'}{q}$$

where p is your price function, q is the quantity function, q' is the derivative of the quantity function.

Note the following:

1. We say that the demand is **elastic** if $E > 1$, is **inelastic** if $E < 1$, and has **unit elasticity** if $E = 1$.

2. If the demand is **inelastic**, raising the price increases revenue.

3. If the demand is **elastic**, lowering the price increases revenue.

4. To determine **maximal revenue price**, find the price p so that $E(p) = 1$, that is, calculate $E(p)$ as a function of p, then set this function equal to 1 and try to solve for p.

Exercise 8. Paint-By-Number:The estimated monthly sales of Mona Lisa paint-by-number sets is given by the formula $q = 100e^{3p^2+p}$, where q is the demand in monthly sales and p is the retail price in dollars.
a. Determine the price elasticity of demand E when the retail price is set at 3 dollars and interpret your answer.
b. At what price will revenue be a maximum?

c. Approximately how many paint-by-number sets will be sold per month at the price in part (b)?

Answers:

a. $E = 51$; the demand is going down 51 percent per 1 percent increase in price at that price level; thus a large price decrease is advised.

b. Revenue is maximized when $p = 0.50$ dollars.

c. Demand would be $100e^{3/4+1/2} \approx 78$ paint-by-number sets per month.

Chapter 3

Suggested Hints

3.1 Section 2.2: Exponential Functions and Exponential Models

Hint for Exercise 1. For f, you can see the slope of all the points are 100. Note that the change in input of consecutive points are 1, so you only look at the change in output (by taking the second output minus the first output). Doing this you also see the slopes of g, h, k are not the same.
Then you see that for g, the factor are all the same 0.2. So this is your base $b = 0.2$. To write equation of the $g(x)$ we then need to find A.
When the input is $x = 0$, the output is $g(x) = g(0) = 4 = A(0,2)^0 = A$ (note that $0.2^0 = 1$). So $g(x) = 4(0,2)^x$.
For h you see that there is a factor of 0.25 and it is different than the other factors 0.5.
For k, the there is one factor of 0.2 and three factors of 0.5.

Hint for Exercise 2. The given data are $(0, 2.56)$ and $(54, 6.40)$ You know that this is **exponential**, so you write $P(t) = Ab^t$. Now we find the base b

first.

$$\frac{6.40}{2.56} = \frac{Ab^{40}}{Ab^0} \text{ [Note the input and output locations]}$$

$$2.5 = b^{40}$$

$$b = (2.5)^{1/40} = 1.02317$$

To find A, plug the b in the data with input $x = 0$ and output $P(0) = 2.56$: $P(0) = A(1.02317)^0 = A$ [note that $1.02317^0 = 1$]

Hint for Exercise 3. Keep in my of the time t as input, we have this data $(0, 360)$ and $(3, 480)$. To find the linear model, find the slope $m = 40$, so $L(t) = 40t + 360$. For the exponential model, similar to example 2. Once you found the model, you see that $L(2) = 440$ is very different (far away from) than the actual profit in the year 1999 ($t = 2$), and $P(2) = 436.075$ is also very far away from that (the actual data from the table for $t = 2$ is 320). SO neither of these models applicable to the data table.

3.2 Section 4.2: Marginal Analysis: Marginal Cost, Marginal Profits

Hint for Exercise 4.

1. Rate of Change, so it is derivatives.
   $C'(x) = 20 - 0.0002x$
   So $C'(50,000) = 20 - 0.0002(50,000) = 10$. The **unit** is **hundred dollars per surface book**

2. Marginal Cost is $C'(x)$ as above. To estimate the 50,001st unit, we need to use the data of the one unit earlier, that is, 50,000. So the estimated cost of the 50,001st surface book is $C'(50,000) = 10$ **hundred dollars**.

3. **Note:** Even though they are both $C'(x)$, they are different things. Please read the question carefully to turn word into notation.

3.3 Section 4.3: The Product and Quotient Rule

just follow the rule, not much thing to hint here.

Hint for Exercise 5. direct calculation

Hint for Exercise 6. 1. $R(t) = Q(t)P(t)$
2. Use the product rule to take derivatives of $R'(t) = Q'(t)P(t) + Q(t)P'(t)$. Then plug in $t = 1$.

3.4 Section 4.4: The Chain Rule

Hint for Exercise 7.

1. We denote $u = (2x^2 + x)$. Then this is the

$$\frac{d}{dx}[u^3] = 3u^2(\frac{du}{dx})$$

The first part is the derivative of f, if we think of $f(u) = u^3$, then derivative of f is $3u^2$, the quantity here is u, and the derivative of the quantity is $(2x^2 + x)' = 4x + 1$. So, remember the chain rule in words, ... is the derivative of f, evaluated at that quantity, **times** the derivative of the quantity is actually $3u^2$ times $4x+1$. Putting this in notation:

$$\frac{d}{dx}[f(u)] = \frac{d}{dx}[u^3] = (3u^2)(4x + 1)$$

Note: Do not forget to replace back u with the original,($u = 2x^2 + x$) so

$$\frac{d}{dx}[(2x^2 + x)^3] = 3(2x^2 + x)^2(4x + 1)$$

2. We denote $u = (x^3 + x)$. Then this is the

$$\frac{d}{dx}[u^{100}] = 100u^{99}(\frac{du}{dx})$$

The first part is the derivative of f, if we think of $f(u) = u^{100}$, then derivative of f is $100u^{99}$, the quantity here is u, and the derivative of the quantity is $(x^3 + x)' = 3x^2 + 1$. So, remember the chain rule in words, ... is the derivative of f, evaluated at that quantity, **times** the derivative of the quantity, is actually $100u^{99}$ times $3x^2 + 1$. Putting this in notation:

$$\frac{d}{dx}[f(u)] = \frac{d}{dx}[u^{100}] = (100u^{99})(3x^2 + 1)$$

Note: Do not forget to replace back u with the original,$(u = x^3 + x)$ so

$$\frac{d}{dx}[(x^3 + x)^{100}] = 100(x^3 + x)^{99}(3x^2 + 1)$$

3. We denote $u = (3x + 1)$. Then this is the

$$\frac{d}{dx}[\sqrt{u}] = (\frac{1}{2u^{1/2}})(\frac{du}{dx})$$

The first part is the derivative of f, if we think of $f(u) = \sqrt{u}$, then derivative of f is $\frac{1}{2u^{1/2}}$, the quantity here is u, and the derivative of the quantity is $(3x + 1)' = 3$. So, remember the chain rule in words, ... is the derivative of f, evaluated at that quantity, **times** the derivative of the quantity is actually $\frac{1}{2u^{1/2}}$ times 3. Putting this in notation:

$$\frac{d}{dx}[f(u)] = \frac{d}{dx}[\sqrt{u}] = (\frac{1}{2u^{1/2}})(3)$$

Note: Do not forget to replace back u with the original,$(u = 3x + 1)$ so

$$\frac{d}{dx}[\sqrt{3x + 1}] = \frac{1}{2(3x + 1)^{1/2}}(3) = \frac{3}{2(3x + 1)^{1/2}}$$

3.5 Section 4.5: The Derivative of Exponential Function

See the Review Material for all Suggested hint.

3.6 Section 5.6: Elasticity

Hint for Exercise 8. a. To find E, we need derivatives of q, so

$$q'(p) = 100[e^{-3p^2+p}]' = 100[e^{-3p^2+p}(\ln e)(-6p+1)] = 100[(e^{-3p^2+p})(-6p+1)].$$

Note that the part $100e^{-3p^2+p}$ is actually same as q. So we can rewrite this as

$$q'(p) = q(-6p+1)$$

Now put back in the Elasticity E formula

$$E(p) = \frac{-q'p}{q} = \frac{-[q(-6p+1)]\,p}{q} = -(-6p+1)p, \text{ [the } q \text{ is canceled]}$$

So

$$E(3) = -(-6(3)+1)(3)) = 51.$$

This demand is elastic, and the demand is going down 51 percent per 1 percent increase in price at that price level; thus a large price decrease is advised.

b.To solve for maximal revenue price, set $E(p) = 1$. So

$$-(-6p+1)p = 1$$

This is the same as

$$6p^2 - p = 1, \text{ [do the distribution]}$$

So we have quadratic equation:

$$6p^2 - p - 1 = 0$$

Solve this using the quadratic formula:

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So we got $p = 0.5$ or $p = -1/3$, and we only take the reasonable price $p = 1$

c. Plug in $p = 1/2$ to the q function:

$$q(1/2) = 100e^{-3/8+1/2} \approx 78 \textbf{ paintings}$$

Chapter 4

Final Remark

Please make sure that you know how to use your calculator, as you are going to need to use power, exponent, the number e, ...etc... I have seen students in our classes did not know how to use their own calculators and this brought a lot of troubles to their answers.

Bibliography