

## Assignment 1.2\_BASIC\_CONCEPTS due 10/01/2021 at 11:59pm PDT

## 1. (1 point)

Match each of the following differential equations with a solution from the list below.

\_\_\_1.  $y'' + 4y' + 4y = 0$

\_\_\_2.  $y'' + y = 0$

\_\_\_3.  $y'' - 4y' + 4y = 0$

\_\_\_4.  $2x^2y'' + 3xy' = y$

→ A.  $y = \cos(x)$  ✗

B.  $y = e^{-2x}$  ✗

C.  $y = \frac{1}{x}$

D.  $y = e^{2x}$

Answer(s) submitted:

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(incorrect)

2. (1 point) Match each differential equation to a function which is a solution.

## FUNCTIONS

A.  $y = 3x + x^2$ ,

B.  $y = e^{-2x}$ ,

C.  $y = \sin(x)$ ,

D.  $y = x^{\frac{1}{2}}$ ,

E.  $y = 4\exp(5x)$ ,

## DIFFERENTIAL EQUATIONS

\_\_\_1.  $xy' - y = x^2$

\_\_\_2.  $2x^2y'' + 3xy' = y$

\_\_\_3.  $y'' + 8y' + 12y = 0$

\_\_\_4.  $y' = 5y$

Answer(s) submitted:

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(incorrect)

## 3. (1 point)

Match the solutions to the differential equations. If there is more than one solution to an equation, select the answer that includes all solutions.

\_\_\_1.  $\frac{dy}{dx} = -7y$

\_\_\_2.  $\frac{d^2y}{dx^2} = -49y$

\_\_\_3.  $\frac{d^2y}{dx^2} = 49y$

\_\_\_4.  $\frac{dy}{dx} = 7y$

A.  $y = e^{-7x}$  or  $y = e^{7x}$

B.  $y = 7\sin(x)$

C.  $y = e^{7x}$

D.  $y = e^{-7x}$

E.  $y = \sin(7x)$

F.  $y = \sin(7x)$  or  $y = 7\sin(x)$

Answer(s) submitted:

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(incorrect)

## 4. (1 point)

Match the solutions to the differential equations. If multiple options could be correct for one of the differential equations, pick the option that includes all solutions.

\_\_\_1.  $y'' - 9y = 0$

\_\_\_2.  $x^2y'' + 2xy' - 2y = 0$

\_\_\_3.  $x^2y'' - 6y = 0$

A.  $y = x^{-2}$  or  $y = x$

B.  $y = e^{4x}$

C.  $y = e^{3x}$  or  $y = e^{-3x}$

D.  $y = x^{-2}$

E.  $y = x^{-1}$

F.  $y = x^6$  or  $y = x^{-5}$

Answer(s) submitted:

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(incorrect)

## 5. (1 point)

Which of the following functions are solutions of the differential equation  $y'' - 5y' - 14y = 0$ ?

• A.  $y(x) = 7x$

• B.  $y(x) = 0$

• C.  $y(x) = -2x$

• D.  $y(x) = e^x$

• E.  $y(x) = e^{-2x}$

• F.  $y(x) = e^{-x}$

• G.  $y(x) = e^{7x}$

Answer(s) submitted:

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(incorrect)

1.2.

1/ A.  $y = \cos x$

$$y' = -\sin x$$

$$y'' = -\cos x$$

$$\cos x + (-\cos x) = 0$$

$$\downarrow y + \downarrow y'' = 0$$

② → A

B.  $y = e^{-2x}$

$$y' = -2e^{-2x}$$

$$y'' = 4e^{-2x}$$

$$4e^{-2x} - 4(-2e^{-2x}) + 4e^{-2x} = 16e^{-2x} \neq 0$$

$$4e^{-2x} + (-2e^{-2x}) \cdot 4 + 4e^{-2x} = 0$$

D:  $y = e^{2x}$

$$y' = 2e^{2x}$$

$$y'' = 4e^{2x}$$

$$3: 4e^{2x} - 4(2e^{2x}) + 4e^{2x} = 0$$

①

C → 4

$$y = \frac{1}{x}$$

$$y' = -\frac{1}{x^2} \rightarrow 3xy' = \frac{-3}{x}$$

$$y'' = \frac{2}{x^3} \rightarrow 2x^2y'' = \frac{4}{x}$$

$$3xy' + 2x^2y'' = \frac{-3}{x} + \frac{4}{x} = \frac{1}{x} = y$$

6. (1 point) The family of functions

$$y = ce^{-2x} + e^{-x}$$

is solution of the equation

$$y' + 2y = e^{-x}$$

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Find the constant  $c$  which defines the solution which also satisfies the initial condition  $y(0) = 2$ .

$c =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

$$x = 0, y = 2$$

$$2 = ce^{-2 \cdot 0} + e^{-0}$$

$$2 = ce^0 + e^0$$

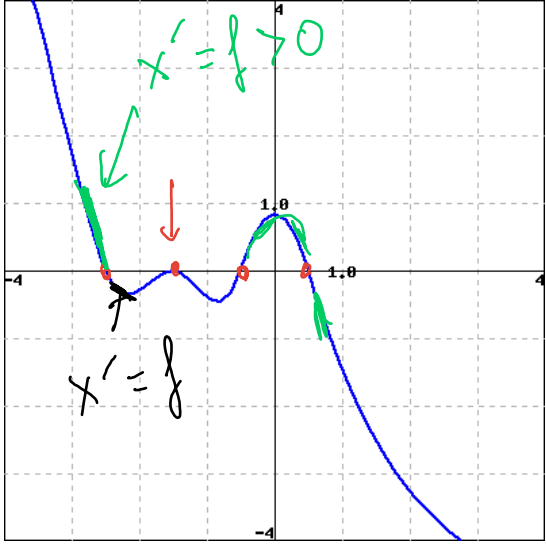
$$2 = c \cdot 1 + 1$$

$$2 = c + 1 \rightarrow c = \underline{1}$$

$$y' = -2ce^{-2x} - e^{-x}$$
$$+ 2y = 2ce^{-2x} + 2e^{-x}$$



1. (1 point) The graph of the function  $f(x)$  is



(the horizontal axis is x.)

Given the differential equation  $x'(t) = f(x(t))$  blue graph  
 List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equilibria are stable, semi-stable, or unstable.

Answer(s) submitted:

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(incorrect)

2. (1 point)

Let  $y(t)$  be a solution of  $\dot{y} = \frac{1}{8}y(1 - \frac{y}{8})$  such that  $y(0) = 16$ . Determine  $\lim_{t \rightarrow \infty} y(t)$  without finding  $y(t)$  explicitly.

$\lim_{t \rightarrow \infty} y(t) =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

3. (1 point) Suppose that a population develops according to the logistic equation

$$\frac{dP}{dt} = 0.15P - 0.0015P^2$$

where  $t$  is measured in weeks.

(a) What is the carrying capacity? \_\_\_\_\_

(b) Is the solution increasing or decreasing when  $P$  is between 0 and the carrying capacity?

(c) Is the solution increasing or decreasing when  $P$  is greater than the carrying capacity?

Answer(s) submitted:

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(incorrect)

\* equilibrium  $x' =$  equation of  $x$   
 When " $x' = 0$ "

Directional Field: field of derivatives

\* Stable: red arrow close to it

- $x = -2.5$
- $x = -1.5$
- $x = -0.5$
- $x = 0.5$

$x' = f$

