Math 31 - Fall 2021 - Week 1 Discussion

1. Think about what it means for an equation to be linear and determine which of the following equations are linear if $a_1, \ldots, a_5, b$ are constants and $x_1, \ldots, x_5, x, y, z$ are variables.

   (a) $y = 2x + 1$

   (b) $x^2 + y^2 = 16$

   (c) $\pi x + \sqrt{3}y = 6 + 2z$

   (d) $a_1x_2 + a_2x_2 + a_3x_3 + a_4x_4^2 + a_5x + 5 = b$

2. What does it mean to be a solution to a system of linear equations? Verify that $x = 1$, $y = 0$, and $z = -1$ is a solution to the system

$$\begin{aligned} x - 2y + z &= 0 \\ 2y - 8z &= 8 \\ 5x - 5z &= 10. \end{aligned}$$

3 variables : (x, y, z)
(5, 6, 7)

3. Solve the system of linear equations

$$\begin{aligned} x - 3y + 4z &= -4 \quad (1) \\ 2x - 7y + 7z &= -8 \quad (2) \\ -4x + 6y - z &= 7 \quad (3) \end{aligned}$$

   without converting it into an augmented matrix (see exercise 5).

4. Find the point of intersection of the lines $x - 5y = 1$ and $3x - 7y = 5$.

5. (If there is time) write the coefficients of the system of linear equations

$$\begin{aligned} x - 3y &= 5 \\ -x + y + 5z &= 2 \\ y + z &= 0 \end{aligned}$$

   in an array where the rows of the array correspond with the equation in which the coefficients appear and the columns correspond with the different variables x, y, and z. For example, the coefficient of x in the first equation will be in the first column and first row of the array. Next, put the constants that appear to the right of the equal sign in the rightmost column of the array separated by a vertical bar and enclose the array in parenthesis so it looks like

$$\left(\begin{array}{ccc|c} a_{1,1} & a_{1,2} & a_{1,3} & b_1 \\ a_{2,1} & a_{2,2} & a_{2,3} & b_2 \\ a_{3,1} & a_{3,2} & a_{3,3} & b_3 \end{array}\right).$$

   You have just created the *augmented matrix* associated with this system of equations.

1) a/ x, y: power 1.
linear.

b/ x, y: power 1
nonlinear.

c/ x, y, z: power 1
linear

d/ x_1, x_2, x_3, x_4, x_5 : variables

x_4 : power of 2 $\longrightarrow$ nonlinear

$\boxed{x = 5,\ y = 6,\ z = 7}$

2/ $(1) - 2 \cdot 0 + (-1) = 0$ ✓

$2(0) - 8(-1) = 8$ ✓

$5(1) - 5(-1) = 10$ ✓

3/ elimination:

$$\begin{array}{r} 2x - 6y + 8z = -8 \\ -\ 2x - 7y + 7z = -8 \\ \hline 0 \quad y + z = 0 \end{array}$$

4/

$$\begin{array}{l} x - 5y = 1 \quad (1) \\ 3x - 7y = 5 \quad (2) \\ \hline 3x - 15y = 3 \\ \hline 0 \quad 8y = 2 \end{array}$$

$\boxed{y = \frac{1}{4}}$

$x = 1 + 5y$

$= 1 + \frac{5}{4} = \frac{9}{4}$

$x - 5y = 1 : \quad x - 1 = 5y \quad \Big| \quad 3x - 5 = 7y$

$\frac{1}{5}x - \frac{1}{5} = y \quad \Big| \quad \frac{3}{7}x - \frac{5}{7} = y$

$\frac{1}{5}x - \frac{1}{5} = \frac{3}{7}x - \frac{5}{7} \rightsquigarrow x = ?$

$$x - 3y = 5$$

$$-x + y + 5z = 2$$

$$y + z = 0$$

Augmented Matrix

$$\left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \\ x & y & z & \end{array}\right)$$

row

left Column

Coefficient

3 variables: x, y, z, w, t

$$\left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right)$$