1. (1 point) Find the particular solution of the differential equation

$$\frac{dy}{dx} + 4y = 9$$

satisfying the initial condition $y(0) = 0$.
Answer: $y =$ ____________________.
Your answer should be a function of x.

Answer(s) submitted:

-

(incorrect)

2. (1 point) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y\cos(x) = 7\cos(x)$$

satisfying the initial condition $y(0) = 9$.
Answer: $y(x)$= ____________________.

Answer(s) submitted:

-

(incorrect)

3. (1 point)
Solve the following initial value problem:

$$t\frac{dy}{dt} + 6y = 6t$$

with $y(1) = 4$.

Find the integrating factor, $u(t) =$ __________,
and then find $y(t) =$ ____________________.

Answer(s) submitted:

-
-

(incorrect)

4. (1 point)
Solve the following initial value problem:

$$\frac{dy}{dt} + 0.7ty = 6t$$

with $y(0) = 3$.
$y =$ ____________________.

Answer(s) submitted:

-

(incorrect)

5. (1 point)
Find the function $y(t)$ that satisfies the differential equation

$$\frac{dy}{dt} - 2ty = 15t^2e^{t^2}$$

and the condition $y(0) = -3$.
$y(t) =$ ____________________.

Answer(s) submitted:

-

(incorrect)

10. (1 point)
A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that, if $n = 0$ or 1, the Bernoulli equation is linear. For other values of n, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

Use an appropriate substitution to solve the equation

$$y' - \frac{6}{x}y = \frac{y^4}{x^6},$$

and find the solution that satisfies $y(1) = 1$.

$y(x) =$ ____________________.

Answer(s) submitted:

-

(incorrect)

$y = (7e^{\sin x} + C) \cdot e^{-\sin x}$ when $x = 0, y = 9$.

$9 = (7e^{\sin 0} + C) \cdot e^{-\sin 0} \Rightarrow C = 2$

2/

2/ $y' + y\cdot\cos x = 7\cos x$ — $f(x)$ — nonhomogeneous

$y' + y\cdot\cos x = 0$ ← homogenous — complementary equation

$y' = -y\cos x$

$\frac{y'}{y} = -\cos x$

(integrate wrt y) = wrt x

$\int \frac{y'}{y}\,dy = \int -\cos x\,dx$

$\ln|y| = -\sin x$

$|y| = e^{-\sin x}$

$y = e^{-\sin x}$ — solution to the homoge

$y = u(x)\cdot e^{-\sin x}$

Find $u(x) = ?$

$y' = u'\cdot e^{-\sin x} + u\cdot(-\sin x)\cdot e^{-\sin x}\cdot(-\cos x)$

$y' = u'\cdot e^{-\sin x} + u\,\sin x\cos x\, e^{-\sin x}$

$y' + y\cos x$

$u' e^{-\sin x} - u\cos x\, e^{-\sin x} + u\cos x\, e^{-\sin x} = 7\cos x$

$u' e^{-\sin x} = 7\cos x$

$u' = \frac{7\cos x}{e^{-\sin x}}$

$u = \int \frac{7\cos x}{e^{-\sin x}}\,dx$

$= \int 7\cos x\, e^{\sin x}\,dx$

$= 7e^{\sin x} + C$

1. (1 point) Find a function y of x such that

$$7yy' = x \text{ and } y(7) = 6.$$

$y =$ ____________.

Answer(s) submitted:

-

(incorrect)

2. (1 point)
Solve the separable differential equation

$$\frac{dx}{dt} = \frac{5}{x},$$

and find the particular solution satisfying the initial condition

$$x(0) = 4.$$

$x(t) =$ ____________.

Answer(s) submitted:

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(incorrect)

3. (1 point)
Solve the separable differential equation

$$\frac{dy}{dx} = \frac{-0.8}{\cos(y)},$$

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$$y(0) = \frac{\pi}{6}.$$

$y(x) =$ ____________.

Answer(s) submitted:

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4. (1 point)
Solve the separable differential equation

$$\frac{dx}{dt} = x^2 + \frac{1}{64},$$

and find the particular solution satisfying the initial condition

$$x(0) = 4.$$

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Answer(s) submitted:

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Find the solution to the differential equation

$$\frac{dz}{dt} = 4te^{7z}$$

that passes through the origin.

$z =$ ____________

Answer(s) submitted:

-

(incorrect)

6. (1 point)
Find the solution to the differential equation

$$\frac{dy}{dx} + \frac{y}{2} = 0,$$

subject to the initial conditions $y(0) = 10$.

$y =$ ____________

Answer(s) submitted:

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$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

P(x) Q(x)

Use an appropriate substitution to solve the equation

$$y' - \frac{6}{x}y = \frac{y^4}{x^6},$$ $\Rightarrow y' + \left(-\frac{6}{x}\right)\cdot y = \frac{1}{x^6} y^4$

and find the solution that satisfies $y(1) = 1$.

$y(x) =$ ________________.

Answer(s) submitted:

-

(incorrect)

$u = \left(\frac{-3}{16}x^{16} + C\right)\cdot x^{-18}$

$y^{-3} = \left(\frac{-3}{16}x^{16} + C\right)\cdot x^{-18}$

raise power $-\frac{1}{3}$

$y = \left(\frac{-3}{16}x^{16} + C\right)^{-\frac{1}{3}}\cdot x^{6}$

$1 = \left(\frac{-3}{16}\cdot 1 + C\right)\cdot 1 \rightarrow C = \frac{19}{16}$

$* \; u = y^{1-4} = y^{-3} \rightarrow u' = -3 \cdot y^{-4} \cdot y'$

$u' = \frac{-3y'}{y^4}$

$y' - \frac{6}{x} y = \frac{1}{x^6} y^4$

$\frac{y'}{y^4} - \frac{6}{x} \cdot y^{-3} = \frac{1}{x^6}$

multiply by -3:

$\frac{-3y'}{y^4} + \frac{18}{x} y^{-3} = \frac{-3}{x^6}$

$u' + \frac{18}{x} u = \frac{-3}{x^3}$ linear u, x

$u' + \frac{18}{x} u = 0$

$\frac{u'}{u} = -\frac{18}{x}$

integrate:

$\ln|u| = -18 \ln|x|$

$u = x^{-18}$ homoge

$u = v \cdot x^{-18}$

Goal: find v

$u' = v' \cdot x^{-18} - 18 v x^{-19}$

$u' + \frac{18}{x} u = \frac{-3}{x^3}$

$v' x^{-18} - 18 v x^{-19} + \frac{18}{x} v \cdot x^{-18} = \frac{-3}{x^3}$

$-18 v x^{-19} + 18 v x^{-19}$

$v' x^{-18} = \frac{-3}{x^3} \Rightarrow v' = -3x^{15}$

$\Rightarrow v = \frac{-3}{16} x^{16} + C$

Step 1:

If not homoge:

Start with homoge

$y' + \frac{6}{t} y = 6$

$y' + \frac{6}{t} y = 0$

$\frac{y'}{y} = \frac{-6}{t}$

integ

$\ln y = -6 \ln t$

$y = t^{-6}$

$y = v \cdot t^{-6}$

$y' = v' \cdot t^{-6} - 6 v t^{-7}$

product rule

plug back to original

can find u

integrated factor

$y = e$

---

1. (1 point) Find a function y of x such that

$$7yy' = x \text{ and } y(7) = 6.$$

$y =$ ______.

Answer(s) submitted:

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(incorrect)

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and find the particular solution satisfying the initial condition

$$x(0) = 4.$$

$x(t) =$ ______.

Answer(s) submitted:

-

(incorrect)

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

$$a = \frac{1}{8}$$

---

5. (1 point)
Find the solution to the differential equation

$$\frac{dz}{dt} = 4te^{7z}$$

that passes through the origin.

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Answer(s) submitted:

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$$\frac{dy}{dx} + \frac{y}{2} = 0,$$

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$y =$ ______

Answer(s) submitted:

-

(incorrect)

4/ $\frac{dx}{dt} = x^2 + \frac{1}{64}$, $x(0) = 4$

$$dx = \left(x^2 + \frac{1}{64}\right) dt$$

$$\frac{1}{x^2 + \frac{1}{64}} dx = dt$$

$$\int \frac{64}{64x^2 + 1} dx = \int dt$$

$$= t + C$$

$$\int \frac{1}{x^2 + \frac{1}{64}} dx$$

$$8 \arctan(8x) = t + C$$

when $t = 0$, $x = 4$

$$8 \arctan(8 \cdot 4) = 0 + C$$

$$C = 8 \arctan(32)$$

$$8 \arctan(8x) = t + 8 \arctan 32$$

$$\arctan(8x) = \frac{t}{8} + \arctan 32$$

$$8x = \tan\left(\frac{t}{8} + \arctan 32\right)$$

$$\boxed{x = \frac{1}{8} \tan\left(\frac{t}{8} + \arctan 32\right)}$$