

Assignment 2.1 LINEAR EQUATIONS due 10/08/2021 at 11:59pm PDT

1. (1 point) Find the particular solution of the differential equation

$$\frac{dy}{dx} + 4y = 9$$

satisfying the initial condition $y(0) = 0$.

Answer: $y =$ _____.

Your answer should be a function of x .

Answer(s) submitted:

(incorrect)

2. (1 point) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \cos(x) = 7 \cos(x)$$

satisfying the initial condition $y(0) = 9$.

Answer: $y(x) =$ _____.

Answer(s) submitted:

(incorrect)

3. (1 point)

Solve the following initial value problem:

$$t \frac{dy}{dt} + 6y = 6t$$

with $y(1) = 4$.

Find the integrating factor, $u(t) =$ _____,

and then find $y(t) =$ _____.

Answer(s) submitted:

(incorrect)

4. (1 point)

Solve the following initial value problem:

$$\frac{dy}{dt} + 0.7ty = 6t$$

with $y(0) = 3$.

$y =$ _____.

Answer(s) submitted:

Handwritten work for problem 1:

$$u' e^{-4x} - 4u e^{-4x} + 4u e^{-4x} = 9$$

$$u' e^{-4x} = 9$$

$$u' = 9 e^{4x}$$

$$u = \frac{9}{4} e^{4x} + C$$

(incorrect)

5. (1 point)

Find the function $y(t)$ that satisfies the differential equation

$$\frac{dy}{dt} - 2ty = 15t^2 e^{t^2}$$

and the condition $y(0) = -3$.

$y(t) =$ _____.

Answer(s) submitted:

(incorrect)

10. (1 point)

A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that, if $n = 0$ or 1 , the Bernoulli equation is linear. For other values of n , the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

Use an appropriate substitution to solve the equation

$$y' - \frac{6}{x}y = \frac{y^4}{x^6},$$

and find the solution that satisfies $y(1) = 1$.

$y(x) =$ _____.

Answer(s) submitted:

(incorrect)

$$1/ \frac{dy}{dx} + 4y = 9$$

$$y(0) = 0$$

$$y' + p(x)y = q \quad \text{want}$$

$$p(x) = 4$$

non-homogeneous

complementary equation: $y' + 4y = 0$

$$p(x) = 4$$

homogeneous

$$y' = -4y : \left(\frac{y'}{y} \right) = -4$$

wrt to y

Integrate both side

wrt x

$$y = \left(\frac{9}{4} e^{4x} - \frac{9}{4} \right) e^{-4x}$$

$$\ln|y| = -4x$$

$$|y| = e^{-4x}$$

$$y = C e^{-4x}$$

$$y' = -4C e^{-4x}$$

$$y' + 4y = 0$$

$$-4C e^{-4x} + 4C e^{-4x} = 0$$

$$y = u \cdot e^{-4x}$$

$$\rightarrow y' = u' \cdot e^{-4x} - u \cdot 4e^{-4x}$$

product rule

Solution of the comp eq

$$y = \left(\frac{9}{4} e^{4x} + C \right) e^{-4x} : 0 = \left(\frac{9}{4} + C \right) \cdot 1$$

Assignment 2.2_SEPARABLE EQUATIONS due 10/08/2021 at 11:59pm PDT

1. (1 point) Find a function y of x such that

$$7yy' = x \text{ and } y(7) = 6.$$

$$y = \underline{\hspace{2cm}}.$$

Answer(s) submitted:

•

(incorrect)

2. (1 point)

Solve the separable differential equation

$$\frac{dx}{dt} = \frac{5}{x},$$

and find the particular solution satisfying the initial condition

$$x(0) = 4.$$

$$x(t) = \underline{\hspace{2cm}}.$$

Answer(s) submitted:

•

(incorrect)

3. (1 point)

Solve the separable differential equation

$$\frac{dy}{dx} = \frac{-0.8}{\cos(y)},$$

and find the particular solution satisfying the initial condition

$$y(0) = \frac{\pi}{6}.$$

$$y(x) = \underline{\hspace{2cm}}.$$

Answer(s) submitted:

•

(incorrect)

4. (1 point)

Solve the separable differential equation

$$\frac{dx}{dt} = x^2 + \frac{1}{64},$$

and find the particular solution satisfying the initial condition

$$x(0) = 4.$$

$$x(t) = \underline{\hspace{2cm}}.$$

Answer(s) submitted:

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(incorrect)

5. (1 point)

Find the solution to the differential equation

$$\frac{dz}{dt} = 4te^{7z}$$

that passes through the origin.

$$z = \underline{\hspace{2cm}}$$

Answer(s) submitted:

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(incorrect)

6. (1 point)

Find the solution to the differential equation

$$\frac{dy}{dx} + \frac{y}{2} = 0,$$

subject to the initial conditions $y(0) = 10$.

$$y = \underline{\hspace{2cm}}$$

Answer(s) submitted:

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$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

$u = y^{1-4} = y^{-3}$

$n = 4$

Use an appropriate substitution to solve the equation

$$y' - \frac{6}{x}y = \frac{y^4}{x^6}, \quad y' - \frac{6}{x}y = \frac{1}{x^6} \cdot y^4$$

and find the solution that satisfies $y(1) = 1$.

$y(x) =$ _____.

Answer(s) submitted:

(incorrect)

$$V' x^{-18} = \frac{-3}{x^6} \implies V' = \frac{-3}{x^6} \cdot x^{18}$$

$$V' = -3x^{12}$$

$$V = -\frac{3}{13}x^{13} + C$$

$$1 = \left(-\frac{3}{13} + C\right) \cdot 1 \implies C = \frac{16}{13}$$

$$U = \left(-\frac{3}{13}x^{13} + C\right) \cdot x^{-18}$$

$$y^{-3} = \left(-\frac{3}{13}x^{13} + C\right) \cdot x^{-18}$$

10/ $u = y^{-3}$: $u' = \frac{du}{dx} = \frac{d}{dx} (y^{-3})$

$$\Rightarrow u' = -3y^{-4} \cdot \frac{dy}{dx} = -\frac{3}{y^4} \cdot y'$$

$$= -3 \cdot \frac{1}{y^4} \cdot y'$$

$$\frac{1}{y^4} \cdot y' - \frac{6}{x} \cdot y^{-3} = \frac{1}{x^6}$$

$\frac{1}{x^6}$: non-homogeneous

$$\frac{u'}{-3} - \frac{6}{x} \cdot u = \frac{1}{x^6}$$

$$u' + \frac{18}{x} u = \frac{-3}{x^6}$$

* by -3

Complementary: $u' + \frac{18}{x} u = 0$

$$u = x^{-18}$$

$$\frac{u'}{u} = -\frac{18}{x}$$

$$\ln |u| = -18 \ln |x|$$

$$= \ln (|x|^{-18})$$

$$u = v x^{-18} \rightarrow u' = v' x^{-18} - 18 v x^{-19}$$

$$v' x^{-18} - 18 v x^{-19} + \frac{18}{x} v x^{-18} = \frac{-3}{x^6}$$

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