

**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$\underline{A} = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

↓  
(1a) Write the reduced row echelon form of  $A$ .

(1b) Write the general solution of the system  $Ax = b$ .

(1c) Write the general solution of the system  $Ax = 0$ .

$[A | b]$  *row reduced*  $\rightarrow$

(2) Find the matrix representing the linear transformations described below.

(2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.

(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?

(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

(4a) Find  $T(u)$ .

(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .

(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

(5a) Find  $AB$ .

(5b) Find  $A^{-1}$ .

$$\left( \begin{array}{cccc|c} 1 & -4 & 8 & 1 & 6 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{4R_2 + R_1 \rightarrow R_1} \left( \begin{array}{cccc|c} 1 & 0 & 0 & 13 & 14 \\ 0 & 1 & -2 & 3 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right)$$

**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

(1a) Write the reduced row echelon form of  $A$ .

(1b) Write the general solution of the system  $Ax = b$ .

(1c) Write the general solution of the system  $Ax = 0$ .

(2) Find the matrix representing the linear transformations described below.

(2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.

(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?

(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

(4a) Find  $T(u)$ .

(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .

(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

(5a) Find  $AB$ .

(5b) Find  $A^{-1}$ .



**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

(1a) Write the reduced row echelon form of  $A$ .

(1b) Write the general solution of the system  $Ax = b$ .

(1c) Write the general solution of the system  $Ax = 0$ .

(2) Find the matrix representing the linear transformations described below.

(2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.

(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?

(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

(4a) Find  $T(u)$ .

(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .

(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

(5a) Find  $AB$ .

(5b) Find  $A^{-1}$ .



**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

(1a) Write the reduced row echelon form of  $A$ .

(1b) Write the general solution of the system  $Ax = b$ .

(1c) Write the general solution of the system  $Ax = 0$ .

(2) Find the matrix representing the linear transformations described below.

(2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.

(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?

(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

(4a) Find  $T(u)$ .

(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .

(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

(5a) Find  $AB$ .

(5b) Find  $A^{-1}$ .



**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

- (1a) Write the reduced row echelon form of  $A$ .  
(1b) Write the general solution of the system  $Ax = b$ .  
(1c) Write the general solution of the system  $Ax = 0$ .

(2) Find the matrix representing the linear transformations described below.

- (2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.  
(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- (3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?  
(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

- (4a) Find  $T(u)$ .  
(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .  
(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

- (5a) Find  $AB$ .  
(5b) Find  $A^{-1}$ .



**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

(1a) Write the reduced row echelon form of  $A$ .

(1b) Write the general solution of the system  $Ax = b$ .

(1c) Write the general solution of the system  $Ax = 0$ .

(2) Find the matrix representing the linear transformations described below.

(2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.

(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?

(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

(4a) Find  $T(u)$ .

(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .

(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

(5a) Find  $AB$ .

(5b) Find  $A^{-1}$ .



**Applied Linear Algebra - MATH 31**  
**UC Riverside - Fall 2021 - Practice Midterm Exam**

---

(1) Let  $A$  be the matrix below and  $b$  be the vector given below.

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$$

- (1a) Write the reduced row echelon form of  $A$ .  
(1b) Write the general solution of the system  $Ax = b$ .  
(1c) Write the general solution of the system  $Ax = 0$ .

(2) Find the matrix representing the linear transformations described below.

- (2a)  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is the rotation by an angle of 90 degrees counterclockwise.  
(2b)  $G : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is given by reflection on the  $x$ -axis.

(3) Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation given by  $T(x) = Ax$  for all  $x$  in  $\mathbb{R}^4$ , where

$$A = \begin{pmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- (3a) Does  $T$  map  $\mathbb{R}^4$  onto  $\mathbb{R}^3$ ?  
(3b) Is  $T$  one-to-one?

(4) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, \quad u = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

- (4a) Find  $T(u)$ .  
(4b) Find all vectors  $x$  in  $\mathbb{R}^2$  such that  $T(x) = b$ .  
(4c) Is  $c$  in the range of  $T$ ?

(5) Let

$$A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & 5 \end{pmatrix}.$$

- (5a) Find  $AB$ .  
(5b) Find  $A^{-1}$ .

