

**1. (1 point)**

Let  $y(t)$  be the solution to  $\dot{y} = 8te^{-y}$  satisfying  $y(0) = 3$ .

(a) Use Euler's Method with time step  $h = 0.2$  to approximate  $y(0.2), y(0.4), \dots, y(1.0)$ .

$k$	$t_k$	$y_k$
0	0	3
1	0.2	_____
2	0.4	_____
3	0.6	_____
4	0.8	_____
5	1.0	_____

(b) Use separation of variables to find  $y(t)$  exactly.

$y(t) =$  \_\_\_\_\_

(c) Compute the error in the approximations to  $y(0.2), y(0.6)$ , and  $y(1)$ .

$|y(0.2) - y_1| =$  \_\_\_\_\_

$|y(0.6) - y_3| =$  \_\_\_\_\_

$|y(1) - y_5| =$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

**3. (1 point)** Use Euler's method with the given step size to estimate  $y(1.4)$  where  $y(x)$  is the solution of the initial-value problem

$$y' = x - xy, \quad y(1) = 1.$$

**1.** Estimate  $y(1.4)$  with a step size  $h = 0.2$ .

Answer:  $y(1.4) \approx$  \_\_\_\_\_

**2.** Estimate  $y(1.4)$  with a step size  $h = 0.1$ .

Answer:  $y(1.4) \approx$  \_\_\_\_\_

Answer(s) submitted:

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(incorrect)

3/p.79:  $\underbrace{14x^2y^3}_{M} dx + \underbrace{21x^2y^2}_{N} dy = 0$

1<sup>st</sup> step: Check for exactness:  $M_y = (14x^2y^3)'_y = 14x^2 \cdot 3y^2 = 42x^2y^2$   
 $N_x = (21x^2y^2)'_x = 21y^2 \cdot 2x = 42xy^2 = \cancel{42x^2y^2}$   
NOT exact:  $42xy^2 \neq 42x^2y^2$

20/p.80:  $\underbrace{(y^3-1)e^x}_{M} dx + \underbrace{3y^2(e^x+1)}_{N} dy = 0, y(0) = 0$

1<sup>st</sup> step: exactness?  $M_y = (y^3-1)e^x = [e^x y^3 - e^x]'_y = 3y^2 e^x$  } equal  
 $N_x = 3y^2(e^x+1) = [3y^2 e^x + 3y^2]'_x = 3y^2 e^x$  } EXACT

2<sup>nd</sup> step:  $F(x,y) = \int M dx = \int (y^3-1)e^x dx = (y^3-1) \int e^x dx$

$F(x,y) = (y^3-1)e^x + g(y)$

3<sup>rd</sup> step: find  $g(y)$ .  $[F(x,y)]'_y = N$

$[y^3 e^x - e^x + g(y)]'_y = N$

$\cancel{3y^2 e^x} - 0 + g'(y) = 3y^2(e^x+1) = \cancel{3y^2 e^x} + 3y^2$

$g'(y) = 3y^2 \Rightarrow g(y) = \int 3y^2 dy$

$g(y) = y^3 + C$

$F(x,y) = (y^3-1)e^x + y^3 + C = 0$

$y(0) = 0$ : when  $x=0, y=0$ :  $(0^3-1)e^0 + 0^3 + C = 0$

$-1 + C = 0$

$C = 1$

$(y^3-1)e^x + y^3 + 1 = 0$

## Assignment 4.1 Growth and Decay due 11/06/2021 at 11:59pm PDT

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1. (1 point)

A continuous annuity with withdrawal rate  $N = \$1,400/\text{year}$  and interest rate  $r = 5$

(a) When will the annuity run out of funds if  $P_0 = \$23,000$ ?

The annuity runs out after approximately \_\_\_\_\_ years.

*Answer to the nearest whole year.*

(b) Which initial deposit  $P_0$  yields a constant balance?  $P_0 =$   
\$ \_\_\_\_\_

*Answer(s) submitted:*

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(incorrect)

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The instantaneous rate of change of the value of a certain investment ( $P$ ) is proportional to its value. That is to say  $\frac{dP}{dt} = rP$ .

If  $r = 7$  and  $P(0) = 3500$ :

$P(t) =$  \_\_\_\_\_.

*Answer(s) submitted:*

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(incorrect)



## Assignment 4.2\_Cooling and Mixing due 11/06/2021 at 11:59pm PDT

1. (1 point) A tank contains 100 kg of salt and 2000 L of water. Pure water enters a tank at the rate 6 L/min. The solution is mixed and drains from the tank at the rate 3 L/min.

(a) What is the amount of salt in the tank initially?

amount = \_\_\_\_\_ (kg)

(b) Find the amount of salt in the tank after 5 hours.

amount = \_\_\_\_\_ (kg)

(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)

concentration = \_\_\_\_\_ (kg/L)

Answer(s) submitted:

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(incorrect)

2. (1 point) A tank contains 100 kg of salt and 2000 L of water. A solution of a concentration 0.025 kg of salt per liter enters a tank at the rate 7 L/min. The solution is mixed and drains from the tank at the same rate.

a.) What is the concentration of our solution in the tank initially?

concentration = \_\_\_\_\_ (kg/L)

b.) Find the amount of salt in the tank after 4.5 hours.

amount = \_\_\_\_\_ (kg)

c.) Find the concentration of salt in the solution in the tank as time approaches infinity.

concentration = \_\_\_\_\_ (kg/L)

Answer(s) submitted:

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(incorrect)

