Math 31 - Fall 2021 - Week 2 - Discussion 3

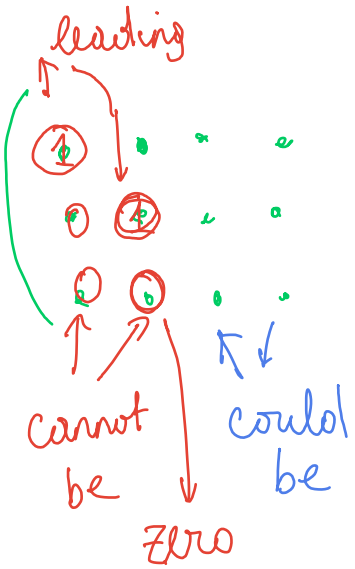
1. What does it mean for a matrix to be in echelon form? What does it mean for a matrix to be in reduced row echelon form? What is a pivot?

2. Convert the matrix
$$\begin{pmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{pmatrix}$$
into echelon form and then into reduced row echelon form.

3. Suppose an augmented matrix has a pivot in every row and every column except the rightmost column. Is the system of linear equations corresponding to such a matrix consistent? If so, how many solutions does it have?

4. Suppose an augmented matrix has a pivot in its rightmost column. Is the system corresponding to this matrix consistent? If so, how many solutions does it have?

5. Suppose an augmented matrix has a pivot in every row but there are three columns that do not have pivots. Is the system of equations corresponding to this matrix consistent? If so, how many solutions does it have?



pivot position $\leftrightarrow$ leading number

correspond to

pivot column

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2/

$$\begin{pmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{pmatrix} \underset{R_1 \leftrightarrow R_3}{\sim} \begin{pmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ \boxed{3} & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{pmatrix}$$

$$R_2 \leftrightarrow R_1 - R_2 \quad \begin{pmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & -2 & 4 & -4 & -2 & 6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{pmatrix} \underset{R_1 \leftrightarrow \frac{1}{3}R_3}{\sim} \begin{pmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & -2 & 4 & -4 & -2 & 6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{pmatrix}$$

$$R_2 \leftrightarrow \tfrac{1}{2}R_2 \quad \begin{pmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & -1 & 2 & -2 & -1 & 3 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{pmatrix} \underset{R_2 \leftrightarrow -1 \cdot R_2}{\longleftrightarrow} \begin{pmatrix} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & \boxed{3} & -6 & 6 & 4 & -5 \end{pmatrix}$$

$$R_3 \leftrightarrow -3R_2 + R_3 \quad \begin{pmatrix} \boxed{1} & -3 & 4 & -3 & 2 & 5 \\ 0 & \boxed{1} & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{pmatrix} \underset{R_1 \leftrightarrow 3R_2 + R_1}{\sim} \begin{pmatrix} 1 & 0 & -2 & 3 & 5 & -4 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & \boxed{1} & 4 \end{pmatrix}$$

$$R_2 \leftrightarrow R_2 - R_3 \quad \begin{pmatrix} 1 & 0 & -2 & 3 & 5 & -4 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{pmatrix} \underset{R_1 \leftrightarrow R_1 - 5R_3}{\longleftrightarrow}$$

$$\begin{array}{c} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right) \\ \begin{array}{ccccc} x & y & z & t & w \end{array} \end{array}$$

not have pivot

z, t are FREE

$\boxed{w = 4}$ → consistent

$y - 2z + 2t = -7$

(2) $\boxed{y = 2z - 2t - 7}$

$x - 2z + 3t = -24$

(3) $\boxed{x = 2z - 3t - 24}$

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5/

"free"

$$\left(\begin{array}{ccc|ccc|c} \boxed{1} & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 & \boxed{9} \\ 0 & 0 & \boxed{1} & \cdot & \cdot & \cdot & 0 & 0 & \boxed{8} \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & \boxed{1} & 0 & \boxed{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} & \boxed{6} \end{array}\right)$$

consistent

infinite

(not unique)

3/

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \\ \begin{array}{ccc} x & y & z \end{array} \end{array}$$

$\longrightarrow x = 7$

$\longrightarrow y = 5$

$\longrightarrow z = 3$

0

right most column

4/

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 5 \end{array}\right)$$

pivot

no solution

not consistent

$0x + 0y + 0z = 5$:(