Math 31 - Fall 2021 - Discussion 6

1. What does it mean to say that a vector w is a linear combination of vectors $v_1, \ldots, v_n$? Write a system of equations that can be used to determine whether w is a linear combination of the vectors $v_1, \ldots, v_n$. Give an example illustrating these concepts in $\mathbb{R}^2$ or $\mathbb{R}^3$.

2. What does it mean to say that some vectors $v_1, \ldots, v_n$ are linearly independent? What does it mean to say that some vectors $v_1, \ldots, v_n$ are linearly dependent? Write a system of equations that can be used to determine whether the vectors $v_1, \ldots, v_n$ are linearly independent or linearly dependent.

3. Determine whether the following set of vectors is linearly independent.

$$v_1 = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} 2 \\ 0 \\ 8 \end{pmatrix} \quad v_3 = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

4. Determine if the columns of the matrix

$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$

are linearly independent.

5. Determine by inspection if each of the given sets is linearly independent.

(a) $\begin{pmatrix} 1 \\ 7 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}.$

(b) $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 8 \end{pmatrix}.$

(c) $\begin{pmatrix} -2 \\ 4 \\ 6 \\ 10 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \\ -9 \\ 15 \end{pmatrix}.$

6. Let's say we have a set of four vectors $\{v_1, v_2, v_3, v_4\}$ in $\mathbb{R}^3$. Can they be linearly independent? Why or why not?

1/ $w = x_1 v_1 + x_2 v_2 + \dots + x_n v_n$

$x_1, \dots, x_n \in \mathbb{R}$

$v_1, \dots, v_n$: vectors.

$$\begin{pmatrix} \vdots & \vdots & \vdots & & \vdots \\ v_1 & v_2 & v_3 & \cdots & v_n \\ \vdots & \vdots & \vdots & & \vdots \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \vdots \\ w \\ \vdots \end{pmatrix}$$

you can find $x_1, \dots, x_n$

cannot be <u>zeros</u>

<u>at the same</u> time.

$$\begin{array}{c} \begin{pmatrix} 1 & 5 & 0 \\ 2 & 6 & 1 \\ 3 & 7 & 2 \end{pmatrix} \\ \uparrow \quad \uparrow \quad \uparrow \\ v_1 \quad v_2 \quad v_3 \end{array} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \\ \uparrow \\ w \end{array}$$

$v_1, \dots, v_n$ <u>indep</u>

2/
$$\begin{pmatrix} \vdots & \vdots & \vdots & & \vdots \\ v_1 & v_2 & v_3 & \cdots & v_n \\ \vdots & \vdots & \vdots & & \vdots \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

If $x_1 = x_2 = x_3 = \dots x_n = 0$

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3/ $\begin{pmatrix} 1 & 2 & 4 \\ 6 & 0 & 1 \\ 0 & 8 & 1 \end{pmatrix}$

$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 8 & 1 \\ 6 & 0 & 1 \end{pmatrix} \xrightarrow{\left(-\frac{1}{6}\right) R_3 + R_1 \;\leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 8 & 1 \\ 0 & 2 & \frac{23}{6} \end{pmatrix}$

into 0

into 0

$\xrightarrow{R_2 - 4R_3 \;\leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 8 & 1 \\ 0 & 0 & -\frac{43}{3} \end{pmatrix}$

$x_1 + 2 \cdot 0 + 4 \cdot 0 = 0 \Rightarrow x_1 = 0$

$8x_2 + 0 = 0 \Rightarrow x_2 = 0$

$-\frac{43}{3} x_3 = 0 \Rightarrow x_3 = 0$

$1 - \frac{4 \cdot 23}{6} = 1 - \frac{2 \cdot 23}{3} = \frac{3 - 46}{3} = \frac{-43}{3}$

B/c all $x_1 = x_2 = x_3 = 0$

so v_1, v_2, v_3 is lin indep.

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4/ $\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix} \xrightarrow[R_1 \leftrightarrow R_3]{} \begin{bmatrix} 5 & 8 & 0 \\ 1 & 2 & -1 \\ 0 & 1 & 4 \end{bmatrix}$

turn into 0

$\xrightarrow[-5R_2 + R_1 \to R_2]{} \begin{bmatrix} 5 & 8 & 0 \\ 0 & -2 & 5 \\ 0 & 1 & 4 \end{bmatrix}$

$\xrightarrow[2R_3 + R_2 \to R_3]{} \left[\begin{array}{ccc|c} 5 & 8 & 0 & 0 \\ 0 & -2 & 5 & 0 \\ 0 & 0 & 13 & 0 \end{array}\right]$

$13x_3 = 0 \Rightarrow x_3 = 0$

$-2x_2 + 5 \cdot 0 = 0 \Rightarrow x_2 = 0$

$5x_1 = 0 \Rightarrow x_1 = 0$

column of the matrix is lin indep.

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$n = 3$

(a) $\overset{v_1}{\begin{pmatrix} 1 \\ 7 \\ 6 \end{pmatrix}}, \overset{v_2}{\begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix}}, \overset{v_3}{\begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}}, \overset{v_4}{\begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}}$.

v_1, v_2, v_3, v_4 in $\mathbb{R}^3$

p

(b) $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 8 \end{pmatrix}$.

$\vec{0}$ in $\mathbb{R}^3$

(c) $\underbrace{\begin{pmatrix} -2 \\ 4 \\ 6 \\ 10 \end{pmatrix}}_{v_1}, \underbrace{\begin{pmatrix} 3 \\ -6 \\ -9 \\ 15 \end{pmatrix}}_{v_2}$.

in $\mathbb{R}^4$

Theorem 8: (p. 60): linearly dependence

$\{v_1, v_2, \ldots, v_p\}$ in $\mathbb{R}^n$ is $\boxed{\text{lin dep}}$

if $p > n$ $[4 > 3]$

6. Let's say we have a set of four vectors $\{v_1, v_2, v_3, v_4\}$ in $\mathbb{R}^3$. Can they be linearly independent? Why or why not?

NO

c): indep.

$6 = \left(-\frac{2}{3}\right) \cdot (-9)$ true ?

$10 = \left(-\frac{2}{3}\right) \cdot (15)$ NOT true

Theorem 9: (p. 60)

$\{v_1, \ldots, v_p\}$

in $\mathbb{R}^n$

contains vector zero

Then: $\boxed{\text{lin dep}}$

(c) If dep:

$v_1 = x \cdot v_2$

$-2 = x \cdot (3)$

$\Rightarrow x = -\frac{2}{3}$.

$4 = \left(-\frac{2}{3}\right) \cdot (-6)$:)