Math 31 - Fall 2021 - Discussion 9

1. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -1 & -2 \\ 2 & 3 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

Determine if the following operations are defined and, if so, compute them.

(a) $A + B$

(b) $A + C$

(c) $3C$

(d) $-2A - B$

2. Compute AB where

$$A = \begin{pmatrix} 2 & 3 \\ 1 & -5 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{pmatrix}.$$

3. Is matrix multiplication commutative? Compute AB and BA where

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

4. If A is a 5×3 matrix and the matrix product AB is a 5×7 matrix, what is the size of B?

5. Find a matrix A corresponding with a horizontal stretch by a factor of 2 and a matrix B that corresponds with reflection across the vertical axis then find the product AB. Describe the linear transformation corresponding with the product AB.

1/a/ A : is 2 by 3

B is 2 by 3

A + B : same dimension

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

you can add or subtract $\Rightarrow$ defined

b/ A is 2 by 3

C : 2 by 2

not same dimension

undefined

c/ $3C = 3\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ -3 & 0 \end{pmatrix}$

d/ $-2A - B$

$$-2\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix} - \begin{pmatrix} 0 & -1 & -2 \\ 2 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -2 & -4 & -6 \\ 2 & 4 & 6 \end{pmatrix} - \begin{pmatrix} 0 & -1 & -2 \\ 2 & 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -3 & -4 \\ 0 & 1 & 2 \end{pmatrix}$$

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2/ $A = \begin{pmatrix} 2 & 3 \\ 1 & -5 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{pmatrix}$

(2) by 2 — 2 by (3)

rows — column — rows — column

AB size 2 by 3

rows — column

$$\begin{pmatrix} 2 & 3 \\ 1 & -5 \end{pmatrix}\begin{pmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 21 \\ -1 & 13 & -9 \end{pmatrix}$$

$2 \cdot 3 + 3 \cdot (-2) = 0$

$2 \cdot 6 + 3 \cdot 3 = 21$

$1 \cdot 4 + -5 \cdot 1 = -1$

$1 \cdot 3 + -5 \cdot (-2) = 13$

$1 \cdot 6 + -5 \cdot 3 = -9$

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A is $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, B is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 2 by 2 * 2 by 2 = 2 by 2

$AB = \begin{pmatrix} ? \end{pmatrix} \stackrel{?}{=} BA = \begin{pmatrix} \end{pmatrix}$

AB

$AB = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$1 \cdot 0 + 1 \cdot 1 = 0 + 1 = 1$

$1 \cdot 1 + 1 \cdot 0 = 1 + 0 = 1$

$0 \cdot 0 + 1 \cdot 1 = 0 + 1 = 1$

$0 \cdot 1 + 1 \cdot 0 = 0 + 0 = 0$

$BA = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$0 \cdot 1 + 1 \cdot 0 = 0$

$0 \cdot 1 + 1 \cdot 1 = 1$

$1 \cdot 1 + 0 \cdot 0 = 1$

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$AB \neq BA$: <u>not</u> commutative

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4 bonus: A is 3 by 5

B is 7 by 3

A B

(3 by 5) * (7 by 3)

5 and 7 ☹ : can NOT multiply.

B A

(7 by 3) * (3 by 5)

3 and 3 ☺ : can multiply

4/ A is 5 by 3

B is _ by _

~~AB~~ is 5 by 7

A B AB

(5 by 3) * (3 by 7) = (5 by 7)

3 = row of B

3 = column of A

7 = column

So B is 3 by 7.

A B

5 3 5

3 7 7

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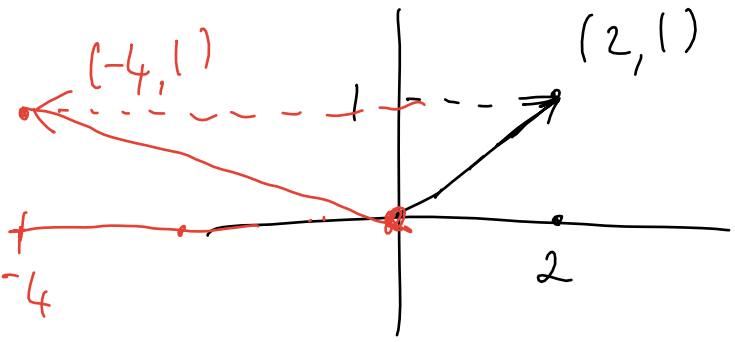
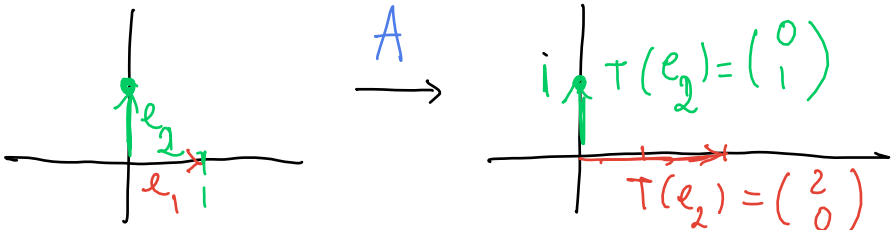


$(2,1) \longmapsto (-4,1)$

A: horizontal stretch by 2



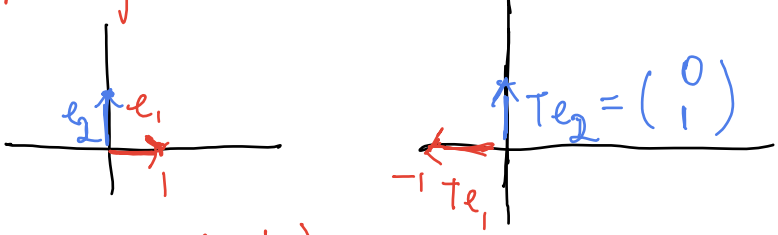
$A=\begin{pmatrix} | & | \\ Te_1 & Te_2 \\ | & | \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$e_2=\begin{pmatrix}0\\1\end{pmatrix} \Rightarrow Te_2=\begin{pmatrix}0\\1\end{pmatrix}$

$e_1=\begin{pmatrix}1\\0\end{pmatrix} \Rightarrow Te_1=\begin{pmatrix}2\\0\end{pmatrix}$

$AB=\begin{pmatrix}2&0\\0&1\end{pmatrix}\begin{pmatrix}-1&0\\0&1\end{pmatrix}=\begin{pmatrix}-2&0\\0&1\end{pmatrix}$

B: reflection across vertical



$Te_1=\begin{pmatrix}-1\\0\end{pmatrix}$

$B=\begin{pmatrix} | & | \\ Te_1 & Te_2 \\ | & | \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$AB\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}-2&0\\0&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=$

$\begin{pmatrix}-2x_1\\x_2\end{pmatrix}$ stretch by -2 in the "x"-direction.