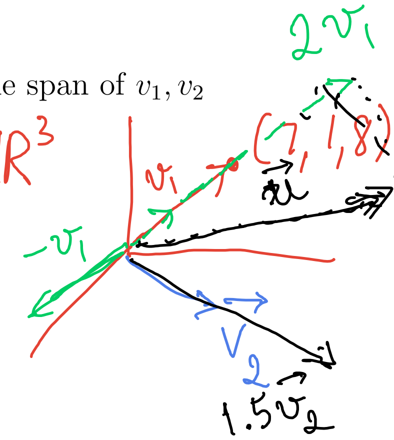
Math 31 - Fall 2021 - Week 3 - Discussion 1

1. What does it means to be in a span of (v_1, v_2)? List 5 vectors in the span of v_1, v_2 where:

$$v_1 = \begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix}$$

3 coor : $\mathbb{R}^3$



and

$$v_2 = \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix}$$

2. For what value of h is b in the spanned by a_1 and a_2 where

$$a_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

and

$$a_2 = \begin{pmatrix} -6 \\ 7 \\ 2 \end{pmatrix}$$

and

$$\boxed{b = \begin{pmatrix} 5 \\ -10 \\ h \end{pmatrix}}$$

in span of (v_1, v_2)

3. Given the following matrix

$$A = \begin{pmatrix} 1 & 0 & -6 \\ 0 & 2 & -2 \\ -2 & 4 & 2 \end{pmatrix}$$

and vector

$$b = \begin{pmatrix} 5 \\ 1 \\ -5 \end{pmatrix}$$

Denote a_1, a_2, a_3 to be the columns of A. Let W to be the span of a_1, a_2, a_3. Is b in W? How many vectors are in W? Show that a_1 is in W.

4. Compute the product of Ax, and if the product is undefined, explain why

(a)

$$\begin{pmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$$

span = linear combination:

$= c_1 v_1 + \underbrace{c_2 v_2}$

$\vec{u} \in span(v_1, v_2)$

belongs to

$\vec{u} = 2v_1 + 1.5v_2$

2

$\frac{1}{2}$

$-1v_1$

$$2\underbrace{\begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix}}_{v_1} + 3\begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 14 \\ 2 \\ 16 \end{pmatrix} + \begin{pmatrix} -18 \\ 12 \\ 0 \end{pmatrix}$$

$$\boxed{\vec{u}} = \begin{pmatrix} -4 \\ 14 \\ 16 \end{pmatrix}$$

$$-1\begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix} + 2\begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ -1 \\ -8 \end{pmatrix} + \begin{pmatrix} -12 \\ 8 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -19 \\ 7 \\ -8 \end{pmatrix} \Rightarrow \vec{v}$$

v_1

$$1\begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix} + 0\begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix}$$

v_2

$$0\begin{pmatrix} 7 \\ 1 \\ 8 \end{pmatrix} + 1\begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

5. (a) Write matrix Ax as a vector equation:

$$\begin{pmatrix} -5 & -3 \\ -4 & 6 \\ -4 & 7 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -27 \\ 12 \\ 16 \\ 3 \end{pmatrix}$$

(b) Write the vector equation as a matrix:

$$x_1 \begin{pmatrix} -4 \\ -1 \\ -5 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ -6 \\ 3 \\ -4 \end{pmatrix} + x_3 \begin{pmatrix} -5 \\ 3 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 8 \\ 7 \end{pmatrix}$$

4.

$$A = \begin{pmatrix} 1 & 0 & -6 \\ 0 & 2 & -2 \\ -2 & 4 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 1 \\ -5 \end{pmatrix}$$

$$\begin{array}{ccc} a_1 & a_2 & a_3 \end{array}$$

a_1, a_2, a_3 columns of A

$W = \text{span}(a_1, a_2, a_3)$ b in W?

How many vectors in $\underline{\underline{W}}$? Show a_1 in W

$\boxed{b \text{ is in } W}$

- $c_1 a_1 + c_2 a_2 + c_3 a_3$: infinite
- $a_1 = 1 \cdot a_1 + 0 a_2 + 0 a_3$ is in span a_1, a_2, a_3

$$c_1 a_1 + c_2 a_2 + c_3 a_3 = b$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -6 & 5 \\ 0 & 2 & -2 & 1 \\ -2 & 4 & 2 & -5 \end{array}\right) \xrightarrow{2R_1 + R_3 \to R_3} \sim \begin{pmatrix} 1 & 0 & -6 & 5 \\ 0 & 2 & -2 & 1 \\ 0 & 4 & -10 & 5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -6 & 5 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & -6 & 3 \end{pmatrix} \quad -2R_2 + R_3 \to R_3$$

$-6c_3 = 3 \Rightarrow \boxed{c_3 = -\frac{1}{2}}$

$2c_2 - 2c_3 = 1 : 2c_2 = 1 + 2c_3 = 1 + -1 = 0$

$\boxed{c_2 = 0}$

$c_1 - 6c_3 = 5 : c_1 = 5 + 6\left(-\frac{1}{2}\right) = 2$

$b = 2a_1 + 0a_2 - \frac{1}{2}a_3$ is in W.

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4. Compute the product of Ax, and if the product is undefined, explain why

(a)

$$\underbrace{\overset{A}{\begin{pmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{pmatrix}}}_{3 \text{ by } 2} \underbrace{\overset{x}{\begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}}}_{3 \text{ by } 1}$$

No

row column

"same" 3 rows by 1

NOT same: cannot

2) $b = c_1 a_1 + c_2 a_2$

$$\begin{pmatrix} 5 \\ -10 \\ h \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} -6 \\ 7 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -10 \\ h \end{pmatrix} = \begin{pmatrix} c_1 \\ 2c_1 \\ -1c_1 \end{pmatrix} + \begin{pmatrix} -6c_2 \\ 7c_2 \\ 2c_2 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -10 \\ h \end{pmatrix} = \begin{pmatrix} c_1 - 6c_2 \\ 2c_1 + 7c_2 \\ -c_1 + 2c_2 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & -6 & 5 \\ 2 & 7 & -10 \\ -1 & 2 & h \end{array}\right)$$

$R_3 \leftrightarrow R_1 + R_3$

$$\begin{pmatrix} 1 & -6 & 5 \\ 2 & 7 & -10 \\ 0 & -4 & 5+h \end{pmatrix}$$

$R_2 \leftrightarrow -2R_1 + R_2$

$$\begin{pmatrix} 1 & -6 & 5 \\ 0 & 19 & -20 \\ 0 & -4 & 5+h \end{pmatrix}$$

$$c_1 - 6c_2 = 5$$
$$2c_1 + 7c_2 = -10$$
$$-c_1 + 2c_2 = h$$

$$19c_2 = -20$$

$$c_2 = \frac{-20}{19}$$

$$-4c_2 = 5 + h$$

$$-4\left(\frac{-20}{19}\right) = 5 + h$$

$$h = \frac{80}{19} - \frac{95}{19} = \boxed{\frac{-15}{19}}$$

3 by 2   2 by 1 $\rightsquigarrow$ 3 by 1

same

(b)
$$\begin{pmatrix} \boxed{6 \quad 5} \\ -4 & -3 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 2 \\ -3 \end{matrix}} \end{pmatrix}$$

- $6 \cdot 2 + 5 \cdot (-3) = -3$
- $-4 \cdot 2 + -3(-3) = 1$
- $7 \cdot 2 + 6 \cdot (-3) = -4$

5. (a) Write matrix Ax as a vector equation:

$$\begin{pmatrix} \boxed{-5 \quad -3} \\ -4 & 6 \\ -4 & 7 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 3 \\ 4 \end{matrix}} \end{pmatrix} = \begin{pmatrix} -27 \\ \boxed{12} \\ 16 \\ 3 \end{pmatrix}$$

$3 \cdot (-5) + \ldots$

$3 \cdot (-4) + \ldots$

$3 \cdot (-4) + \ldots$

$3 \cdot (-3)$

(b) Write the vector equation as a matrix:

$$x_1 \begin{pmatrix} -4 \\ -1 \\ -5 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ -6 \\ 3 \\ -4 \end{pmatrix} + x_3 \begin{pmatrix} -5 \\ 3 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 8 \\ 7 \end{pmatrix}$$

5(a)
$$3 \begin{pmatrix} -5 \\ -4 \\ -4 \\ -3 \end{pmatrix} + 4 \begin{pmatrix} -3 \\ 6 \\ 7 \\ 3 \end{pmatrix}$$

5/b/

$$\overset{A}{\begin{pmatrix} -4 & -3 & -5 \\ -1 & -6 & 3 \\ -5 & 3 & -2 \\ 0 & -4 & 5 \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 8 \\ 7 \end{pmatrix}$$

" x_1 "