Math 31 - Fall 2021 - Week 3 - Discussion 2 - Day 5

1. Do the columns of A spans $\mathbb{R}^4$? Does equation $Ax = b$ has a solution for each vector b in $\mathbb{R}^4$ where

$$A = \begin{pmatrix} 2 & 1 & -5 & 1 \\ 1 & 0 & -4 & 2 \\ 3 & 1 & -9 & 3 \\ -1 & -9 & -23 & 29 \end{pmatrix}$$

2. Given

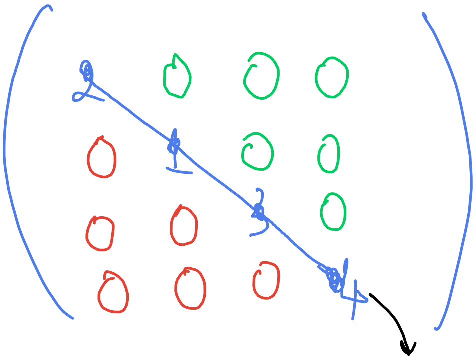
$$u = \begin{pmatrix} -3 \\ -5 \\ -3 \end{pmatrix}$$

and

$$v = \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix}$$

and

$$w = \begin{pmatrix} 24 \\ 1 \\ -12 \end{pmatrix}$$



Final goal:

It can be shown that $-2u - 3v - w = 0$. Use this fact and do **not** use row operation to find x_1, x_2 satisfies:

$$\begin{pmatrix} -3 & -6 \\ -5 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 24 \\ 1 \\ -12 \end{pmatrix}$$

3. (a) Let A be a 3×3 matrix with 2 pivot positions. Does $Ax = 0$ has non-trivial solution and why? Does $Ax = b$ has at least 1 solution for all possible vector b and why?

   (b) Let A be a 3×3 matrix with 3 pivot positions. Does $Ax = 0$ has non-trivial solution and why? Does $Ax = b$ has at least 1 solution for all possible vector b and why?

4. Describe the solution set of $x_1 + 4x_2 - 7x_3$. Then compare it with the solution set of $x_1 + 4x_2 - 7x_3 = -3$

5. Describe all solutions of $Ax = 0$ in a parametric vector form where

$$A = \begin{pmatrix} 1 & 4 & -4 & 7 \\ 0 & 1 & -4 & 4 \end{pmatrix}$$

Theorem 4. (p. 37):

(a) and (c) $\Longrightarrow$ NO

Use (d): Does A have pivot position in every row : NO

$$A = \begin{pmatrix} 2 & 1 & -5 & 1 \\ 1 & 0 & -4 & 2 \\ 3 & 1 & -9 & 3 \\ -1 & -9 & -23 & 29 \end{pmatrix}$$

$-2R_2 + R_1 \leftrightarrow R_2$

$-3R_2 + R_3 \leftrightarrow R_3$

$R_4 \leftrightarrow R_2 + R_4$

$$\begin{pmatrix} 2 & 1 & -5 & 1 \\ 0 & 1 & 3 & -3 \\ 0 & 1 & 3 & -3 \\ 0 & -9 & -27 & 31 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & -5 & 1 \\ 0 & 1 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & -9 & -27 & 31 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & -5 & 1 \\ 0 & 1 & 3 & -3 \\ 0 & -9 & -27 & 31 \\ \boxed{0 \quad 0 \quad 0 \quad 0} \end{pmatrix}$$

R_4 does not have pivot answer.

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2/ $-2u - 3v - w = 0$

$$A = \begin{array}{cc} u & v \\ \hline \end{array} \begin{pmatrix} -3 & -6 \\ -5 & 3 \\ 3 & 2 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$w = \begin{pmatrix} 24 \\ 1 \\ -12 \end{pmatrix}$$

$Ax = w$

$$\begin{pmatrix} u & v \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = w$$

$x_1 u + x_2 v = w$

$-2u + (-3)v = w$

$x_1 = -2, \; x_3 = -3.$

3/a/ A : 3 by 3, has 2 pivot positions.

i/ $Ax = 0$ has non-trivial soln? $\boxed{\text{YES}}$

ii/ $Ax = b$ has at least 1 soln for all b? $\boxed{\text{NO}}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = 0$

$x_2 = 0$

$0x_3 = 0$

$$\boxed{\begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 100 \end{array}}$$

x_3 can be any value

$\boxed{x_3 \text{ is free}}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 7 \end{array}\right)$$

$x_1 = 5$

$x_2 = 6$

$0x_3 = 7 \qquad 0 \overset{?}{=} 7$

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b/ A 3 by 3, 3 pivot positions

(i) $Ax = 0$ has non-trivial soln? NO

(ii) $Ax = b$ has at least 1 soln? YES

$$\left(\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \end{array}\right) \longrightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}$$

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$$\begin{pmatrix} -12x_3 + 9x_4 \\ 4x_3 - 4x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

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   (b) Let A be a 3×3 matrix with 3 pivot positions. Does $Ax = 0$ has non-trivial solution and why? Does $Ax = b$ has at least 1 solution for all possible vector b and why?

$$x_3 \begin{pmatrix} -12 \\ 4 \\ 1 \\ 0 \end{pmatrix} + \cancel{x}_4 \begin{pmatrix} 9 \\ -4 \\ 0 \\ 1 \end{pmatrix}$$

4. Describe the solution set of $x_1 + 4x_2 - 7x_3$ $, 0$. Then compare it with the solution set of $x_1 + 4x_2 - 7x_3 = -3$

5. Describe all solutions of $Ax = 0$ in a parametric vector form where

$$A = \begin{pmatrix} 1 & \textcircled{4} & -4 & 7 \\ 0 & 1 & -4 & 4 \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & 0 & 12 & -9 \\ 0 & \boxed{1} & \underline{-4} & \underline{4} \end{pmatrix}$$

$$x_1 + 12x_3 - 9x_3 = 0$$
$$x_2 - 4x_3 + 4x_4 = 0$$

$$\boxed{x_3, x_4 \text{ free}}$$

$$x_1 = -12x_3 + 9x_4$$
$$x_2 = 4x_3 - 4x_4$$
$$\left.\begin{matrix} x_3 \\ x_4 \end{matrix}\right\} \text{free}$$

$\boxed{x_1 + 4x_2 - 7x_3 = 0 \quad S_1}$

$x_1 + 4x_2 - 7x_3 = -3 : S_2$

$x_1 = -4\textcircled{x_2} + 7\textcircled{x_3}$

free

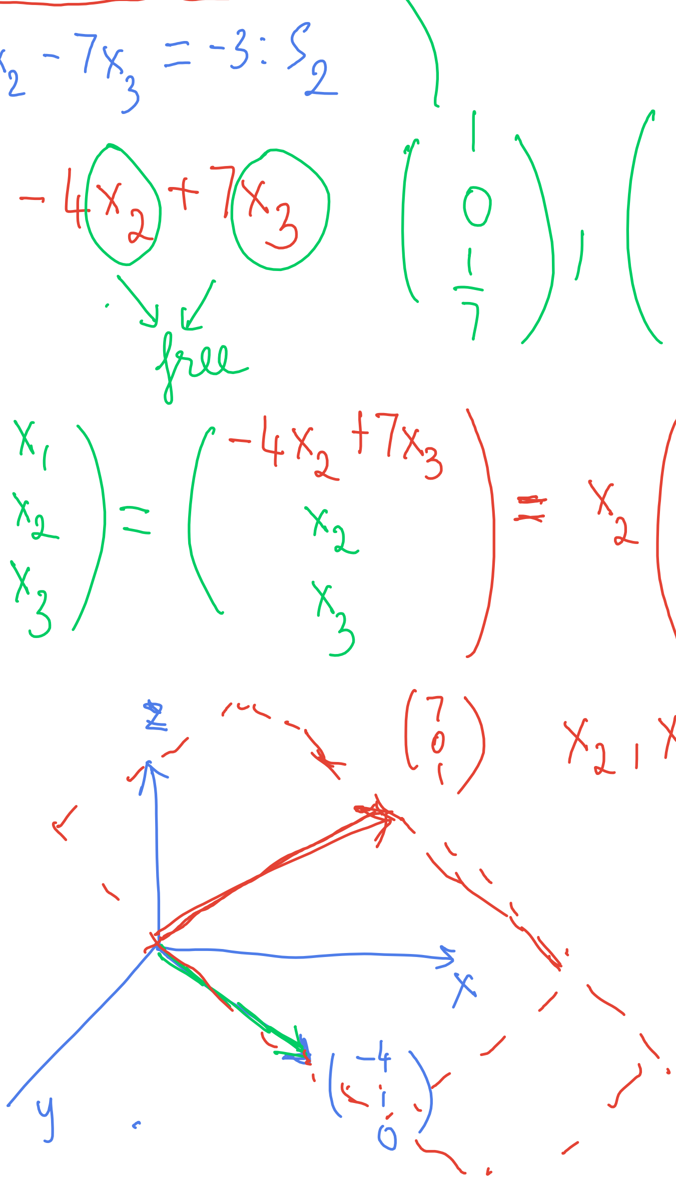
$\begin{pmatrix} 1 \\ 0 \\ \frac{1}{7} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{4}{7} \end{pmatrix}$

$x_1 + 1$

$x_1 = 4x_2 - 7x_3 - 3$

$\vec{x} = \begin{pmatrix} 4x_2 - 7x_3 - 3 \\ x_2 \\ x_3 \end{pmatrix}$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4x_2 + 7x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix}$$



x_2, x_3 are free, any $\mathbb{R}$ number.

$$\vec{x} = x_2 \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$$

$\downarrow$

S_2 : shift of S_1 to the x direction by -3.