

Math 46 - Fall 2021 - Discussion section 002 and 005

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Homework due Oct 22

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## 1 2.4. Transformation of Non Linear Equations into Separable Equations

**Exercise 1.** A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that, if  $n = 0$  or  $1$ , the Bernoulli equation is linear. For other values of  $n$ , the substitution  $u = y^{1-n}$  transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1 - n)P(x)u = (1 - n)Q(x).$$

Use an appropriate substitution to solve the equation

$$xy' + y = 9xy^2,$$

and find the solution that satisfies  $y(1) = -5$ .

**Exercise 2.** A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \quad (*)$$

Observe that, if  $n = 0$  or  $1$ , the Bernoulli equation is linear. For other values of  $n$ , the substitution  $u = y^{1-n}$  transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

Consider the initial value problem

$$xy' + y = -4xy^2, \quad y(1) = -5.$$

- (a) This differential equation can be written in the form (\*) with  $P(x) = ?$ ,  $Q(x) = ?$ , and  $n = ?$
- (b) The substitution  $u = ?$  will transform it into the linear equation  $\frac{du}{dx} + ?u = ?$
- (c) Using the substitution in part (b), we rewrite the initial condition in terms of  $x$  and  $u$ :  $u(1) = ?$
- (d) Now solve the linear equation in part (b), and find the solution that satisfies the initial condition in part (c).  $u(x) = ?$
- (e) Finally, solve for  $y$ .  
 $y(x) = \underline{\hspace{4cm}}$ .

**Exercise 3.** A Bernoulli differential equation is one of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

$n=4$ :  $u = y^{1-4} = y^{-3} = \frac{1}{y^3}$

$$u' = -3y^{-4}y' = \frac{-3y'}{y^4}$$

Observe that, if  $n = 0$  or  $1$ , the Bernoulli equation is linear. For other values of  $n$ , the substitution  $u = y^{1-n}$  transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x)$$

$$\frac{y'}{y^4} - \frac{6}{x} \frac{1}{y^3} = \frac{1}{x^{16}}$$

Use an appropriate substitution to solve the equation

$$\frac{u'}{-3} = \frac{6}{x} u \Rightarrow \frac{u'}{-3u} = \frac{6}{x} \Rightarrow \frac{u'}{u} = \frac{-18}{x}$$

$$y' - \frac{6}{x}y = \frac{y^4}{x^{16}}$$

$$\frac{u'}{-3} - \frac{6}{x}u = \frac{1}{x^{16}}$$

and find the solution that satisfies  $y(1) = 1$ .

$$\ln u = -18 \ln x = \ln x^{-18} \Rightarrow u_1 = x^{-18}$$

$$\frac{u'}{-3} - \frac{6}{x}u = 0 \Rightarrow \frac{u'}{u} = \frac{-3}{6x} = \frac{-18}{x}$$

## 2 Exact

**Exercise 1.** Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function  $F(x,y)$  whose differential,  $dF(x,y)$  is the left hand side of the differential equation. That is, level curves  $F(x,y) = C$  are solutions to the differential equation:

$$(-1x^4 + 2y)dx + (2x + 3y^2)dy = 0$$

First:

$$M_y(x,y) = ?, \text{ and } N_x(x,y) = ?$$

If the equation is not exact, enter *not exact*, otherwise enter in  $F(x,y)$  here \_\_\_\_\_

**Exercise 2.** Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function  $F(x,y)$  whose differential,  $dF(x,y)$  is the left hand side of the differential equation. That is, level curves  $F(x,y) = C$  are solutions to the differential equation:

$$(2xy^2 + 3y)dx + (2x^2y + 3x)dy = 0$$

First:  $M_y(x,y) =$  \_\_\_\_\_ , and  $N_x(x,y) =$  \_\_\_\_\_.

If the equation is not exact, enter *not exact*, otherwise enter in  $F(x,y)$  here \_\_\_\_\_

**Exercise 3.** Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function  $F(x, y)$  whose differential,  $dF(x, y)$  gives the differential equation. That is, level curves  $F(x, y) = C$  are solutions to the differential equation:

$$\frac{dy}{dx} = \frac{x^3 - y}{x + 4y^4}$$

First rewrite as

$$M(x, y) dx + N(x, y) dy = 0$$

where  $M(x, y) = \underline{\hspace{2cm}}$  ,  
and  $N(x, y) = \underline{\hspace{2cm}}$ .

If the equation is not exact, enter *not exact*, otherwise enter in  $F(x, y)$  as the solution of the differential equation here  $\underline{\hspace{2cm}} = C$ .

**Exercise 4.** Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function  $F(x, y)$  whose differential,  $dF(x, y)$  is the left hand side of the differential equation.

That is, level curves  $F(x, y) = C$  are solutions to the differential equation

$$(-4e^x \sin(y) + 2y)dx + (2x + 4e^x \cos(y))dy = 0$$

First, if this equation has the form  $M(x, y)dx + N(x, y)dy = 0$ :

$M_y(x, y) = \underline{\hspace{2cm}}$  , and  $N_x(x, y) = \underline{\hspace{2cm}}$ .

If the equation is not exact, enter *not exact*, otherwise enter in  $F(x, y)$  here  $\underline{\hspace{4cm}}$

**Exercise 5.** The differential equation

$$y - 3y^2 = (y^6 + x) y'$$

can be written in differential form:

$$M(x, y) dx + N(x, y) dy = 0$$

where

$$M(x, y) = \text{_____}, \text{ and } N(x, y) = \text{_____}.$$

The term  $M(x, y) dx + N(x, y) dy$  becomes an exact differential if the left hand side above is divided by  $y^2$ . Integrating that new equation, the solution of the differential equation is \_\_\_\_\_ =  $C$ .