1. (1 point)

It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear** .
Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

[?] 1. $y'' - y + t^2 = 0$
[?] 2. $\frac{dy}{dt} + ty^2 = 0$
[?] 3. $y'' - y + y^2 = 0$
[?] 4. $\frac{d^3y}{dt^3} + t\frac{dy}{dt} + (cos^2(t))y = t^3$

2. (1 point)

Determine which of the following pairs of functions are linearly independent.

[?] 1. $f(t) = e^{\lambda t}\cos(\mu t) \quad , \quad g(t) = e^{\lambda t}\sin(\mu t) \quad , \mu \neq 0$
[?] 2. $f(x) = x^2 \quad , \quad g(x) = 4|x|^2$
[?] 3. $f(\theta) = \cos(3\theta) \quad , \quad g(\theta) = 20\cos^3(\theta) - 15\cos(\theta)$
[?] 4. $f(x) = e^{5x} \quad , \quad g(x) = e^{5(x-3)}$

3. (1 point)

Determine whether the following pairs of functions are linearly independent or not.

[?] 1. $f(t) = t^2 + 4t$ and $g(t) = t^2 - 4t$
[?] 2. $f(t) = t$ and $g(t) = |t|$
[?] 3. $f(x) = e^{4x}$ and $g(x) = e^{4(x-1)}$

2/ $W(f, g) = fg' - gf'$

$= e^{\lambda t} \cos \mu t \cdot \left[\lambda e^{\lambda t} \sin(\mu t) + e^{\lambda t} \mu \cos(\mu t)\right]$

$- e^{\lambda t} \sin(\mu t)$

1. (1 point) Match the following differential equations with their solutions.
The symbols A, B, C in the solutions stand for arbitrary constants.
You must get all of the answers correct to receive credit.

___1. $\frac{d^2y}{dx^2}+9y=0$
___2. $\frac{dy}{dx}=\frac{-2xy}{x^2-3y^2}$
___3. $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+25y=0$
___4. $\frac{dy}{dx}=6xy$
___5. $\frac{dy}{dx}+15x^2y=15x^2$

A. $y=Ae^{-5x}+Bxe^{-5x}$
B. $y=Ae^{3x^2}$
C. $y=Ce^{-5x^3}+1$
D. $3yx^2-3y^3=C$
E. $y=A\cos(3x)+B\sin(3x)$

2. (1 point)
Check by differentiation that $y=4\cos 3t+6\sin 3t$ is a solution to $y''+9y=0$ by finding the terms in the sum:
$y''=$ ____________
$9y=$ ____________
So $y''+9y=$ ______

3. (1 point)
Solve the following differential equation:

$$y''-2y'+2y=0$$

and express your answer in the form

$$ce^{\alpha x}\sin(\beta x+\gamma)$$

Answer: $\alpha=$ ________, $\beta=$ ________.

4. (1 point)
Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}+15\frac{dy}{dt}+56y=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1=$ ______ and $r_2=$ ______

5. (1 point) Find the general solution to the homogeneous differential equation

$$\frac{d^2y}{dt^2}-14\frac{dy}{dt}=0$$

The solution can be written in the form

$$y=C_1e^{r_1t}+C_2e^{r_2t}$$

with

$$r_1<r_2$$

Using this form, $r_1=$ ______ and $r_2=$ ______

6. (1 point)
Find the solution to the boundary value problem:

$$\frac{d^2y}{dt^2}-13\frac{dy}{dt}+36y=0,\quad y(0)=6, y(1)=4$$

The solution is $y=$ ____________________