Math 31 - Fall 2021 - Discussion 14 - November 16

1. The set $\mathcal{B} = \{\overset{v_1}{1 + t^2}, \overset{v_2}{2t - t^2}, \overset{v_3}{1 + t + t^2}\}$ is a basis for $\mathcal{P}_2$ (the vector space of all polynomials of degree less than or equal to 2). Find the coordinate vector of $p(t) = -5 + 11t - 11t^2$ relative to $\mathcal{B}$.

2. Find a basis for the subspace and state its dimension:

$$\left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} : a - 4b + 5c = 0 \right\}$$

3. Determine the dimension of the Nul(A) and Col(A) where

$$A = \begin{pmatrix} 1 & 5 & 3 & 3 & -9 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4. Find the dimension of the subspace spanned by:

$$\left\{ \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, \begin{pmatrix} 22 \\ -39 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 7 \end{pmatrix} \right\}$$

5. Find the dimension of subspace of all vectors in $\mathbb{R}^7$ whose fifth and sixth entries are equal?

1/

$$\begin{array}{c} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -5 \\ 0 & 2 & 1 & 11 \\ 1 & -1 & 1 & -11 \end{array}\right) \\ \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \end{array} \underset{R_1 - R_3 \to R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -5 \\ 0 & 2 & 1 & 11 \\ 0 & 1 & 0 & 6 \end{array}\right)$$

$$\underset{R_2 - 2R_3 \to R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -5 \\ 0 & 2 & 1 & 11 \\ 0 & 0 & \boxed{1} & \boxed{-1} \end{array}\right)$$

- $1 + t^2 = 1 + 0t + t^2$
  $= 1 + 0t + 1t^2$
- $2t - t^2 = 0 + 2t - 1t^2$
- $\underline{1} + \underset{1}{t} + \underset{1}{t^2} =$

$p(t) = -5 + 11t - 11t^2$

$x_3 = -1$ ✓

$x_1 + x_3 = -5$

$x_1 = -5 - x_3$

$= -5 - (-1) = -4$

$2x_2 + x_3 = 11$

$2x_2 = 11 - x_3 = 11 - (-1)$

$= 12$

$\boxed{x_2 = 6}$ ✓

$\begin{bmatrix} -4 \\ 6 \\ -1 \end{bmatrix}$ coordinate of $p(t)$ relative to β.

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2/ $\begin{pmatrix} 4b-5c \\ b \\ c \\ d \end{pmatrix} = b\begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c\begin{pmatrix} -5 \\ 0 \\ 1 \\ 0 \end{pmatrix} + d\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$a = 4b - 5c$

Basis $= \left\{ \begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

3 vectors in basis : so dim of basis is 3.

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3/ * **dimension of Nul(A):** number of free variables in augmented matrix $Ax = 0$

$[A \mid 0]$

x_1 : pivot

$$\left(\begin{array}{ccccc|c} 1 & 5 & 3 & 3 & -9 & 0 \\ 0 & 0 & 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\boxed{x_5 = 0}$

$x_4 + 8x_5 = 0 \Rightarrow \boxed{x_4 = 0}$

$x_1 = -5x_2 - 3x_3$

x_2 and x_3 are free : 2 free variables

So dim of Nul(A) is 2

* **dimension of Col(A):** number of pivot columns in A

$$\begin{pmatrix} \boxed{1} & 5 & 3 & 3 & -9 \\ 0 & 0 & 0 & \boxed{1} & 8 \\ 0 & 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

pivot column

pivot column

pivot column

3 pivot columns in A

$\Rightarrow$ dim of Col(A) = 3.

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4/ dimension $\underline{\underline{\text{is}}}$ maximum number of linearly independent vector in it.

$$\begin{bmatrix} 1 & 3 & 22 & 3 \\ -2 & -5 & -39 & 0 \\ 0 & 1 & 5 & 7 \end{bmatrix} \underset{\substack{2R_1 + R_2 \\ \to R_2}}{\sim} \begin{bmatrix} 1 & 3 & 22 & 3 \\ 0 & 1 & 5 & 6 \\ 0 & 1 & 5 & 7 \end{bmatrix}$$

$$\underset{\substack{R_3 - R_2 \\ \to R_3}}{\sim} \begin{bmatrix} \boxed{1} & 3 & 22 & 3 \\ 0 & \boxed{1} & 5 & 6 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

pivot column   pivot column

pivot column

linearly indep column "pivot columns"!

↓

3 pivot columns

⟹ 3 linearly indep column

⟹ dim of subspace is $\underline{\underline{3}}$.

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5/

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_5 \\ x_7 \end{pmatrix} = x_1 \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{v_1} + x_2 \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{v_2} + x_3 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{v_3}$$

$$+ x_4 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{v_4} + x_5 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}}_{v_5} + x_7 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{v_6}$$

$$\boxed{\text{dim} = 6}$$

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(*) basis of Nul A: must solve $Ax = 0$

$$x_1 \begin{bmatrix} \\ \\ \end{bmatrix} + x_3 \begin{bmatrix} \\ \\ \end{bmatrix} + \ldots$$

basis

(*) rank A: dimension of Col A

= number of pivot (column) in A.

(*) basis for row space of A:

1/ row reduced A into B.

2/ the "non-zero" rows of matrix B.

is the basis for row space of A

(*) basis for column space of A:

the columns corresponding to the pivots

(the pivot columns)

(*) A is m by $\boxed{n}$ matrix (# of column)

dim(Nul A) + rank A = n

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