Math 31 - Fall 2021 - Discussion 11

1. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}.$$

You should get

$$\boxed{A^{-1} = \begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{pmatrix}.}$$

Use this to solve the system of equations $Ax = b$ where

$$b = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

2. Which of the following matrices are invertible? Explain your answers.

$$A = \begin{pmatrix} 16 & -4 \\ -12 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$C = \begin{pmatrix} 3 & 0 & -5 \\ 2 & 0 & 4 \\ -4 & 0 & 8 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ -3 & 8 & 0 \end{pmatrix}$$

3. Let A be a matrix of size $n \times n$ with entries in $\mathbb{R}$. Explain why each of the following statements holds.

   a. If $Ax = 0$ has only trivial solution, then A is row equivalent to the $n \times n$ identity matrix.

   b. If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$.

   c. If A is invertible, then the columns of A^{-1} is linearly independent

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ define by $T(x_1, x_2) = (6x_1 - 8x_2, -6x_1 + 9x_2)$. Show that T is invertible and find the determinant of the standard matrix of T.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\to R_2]{3R_1+R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\to R_3]{-2R_1+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & \boxed{-3} & 8 & -2 & 0 & 1 \end{array}\right) \xrightarrow[\to R_3]{3R_2+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \boxed{-2} & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right)$$

$$\xrightarrow[\to R_1]{R_1+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & \boxed{-2} & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right) \xrightarrow[\to R_2]{R_2+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{array}\right)$$

$$Ax = b$$
$$A^{-1}Ax = A^{-1}b$$
$$Ix = A^{-1}b$$

$$\boxed{x = A^{-1}b}$$

$$\begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 3 \end{pmatrix}$$

Math 31 - Fall 2021 - Discussion 11

1. Find the inverse of the matrix
$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}.$$
You should get
$$A^{-1} = \begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{pmatrix}.$$
Use this to solve the system of equations $Ax = b$ where
$$b = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

2. Which of the following matrices are invertible? Explain your answers.
$$A = \begin{pmatrix} 16 & -4 \\ -12 & 3 \end{pmatrix}$$
$$B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$
$$C = \begin{pmatrix} 3 & 0 & -5 \\ 2 & 0 & 4 \\ -4 & 0 & 8 \end{pmatrix}$$
$$D = \begin{pmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ -3 & 8 & 0 \end{pmatrix}$$

3. Let A be a matrix of size $n \times n$ with entries in $\mathbb{R}$. Explain why each of the following statements holds.
   a. If $Ax = 0$ has only trivial solution, then A is row equivalent to the $n \times n$ identity matrix.
   b. If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$.
   c. If A is invertible, then the columns of A^{-1} is linearly independent

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ define by $T(x_1, x_2) = (6x_1 - 8x_2, -6x_1 + 9x_2)$. Show that T is invertible and find the determinant of the standard matrix of T.

2/ 2 by 2: $\frac{1}{ad-bc}$

- $ad - bc = \det A \neq 0$.
  $16 \cdot 3 - (-12)(-4) = 0$: $\rightarrow$ not invertible
- $3 \cdot 4 - 1 \cdot 2 = 10 \neq 0$: invertible.

Theorem 8 (p. 114):

e: column of C create a linearly independence

$$\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}, \boxed{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}, \begin{pmatrix} -5 \\ 4 \\ 8 \end{pmatrix}$$: dependent

not indep

NOT invertible

c) 3 pivots

$$\begin{pmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ -3 & 8 & 0 \end{pmatrix} \xrightarrow{3R_1 + R_3} \begin{pmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ 0 & -4 & -18 \end{pmatrix}$$

$$\xrightarrow{4R_2 + 5R_3 \to R_3} \begin{pmatrix} 1 & \boxed{-4} & -6 \\ 0 & 5 & 4 \\ 0 & 0 & -74 \end{pmatrix} \xrightarrow{4R_2 + 5R_1 \to R_1} \begin{pmatrix} 1 & 0 & -14 \\ 0 & 5 & 4 \\ 0 & 0 & -74 \end{pmatrix}$$

$16 - 90$ $\quad$ $16 - 30$

3 pivots $\Rightarrow$ invertible

Math 31 - Fall 2021 - Discussion 11

1. Find the inverse of the matrix
$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}.$$
You should get
$$A^{-1} = \begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{pmatrix}.$$
Use this to solve the system of equations $Ax = b$ where
$$b = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

2. Which of the following matrices are invertible? Explain your answers.
$$A = \begin{pmatrix} 16 & -4 \\ -12 & 3 \end{pmatrix}$$
$$B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$
$$C = \begin{pmatrix} 3 & 0 & -5 \\ 2 & 0 & 4 \\ -4 & 0 & 8 \end{pmatrix}$$
$$D = \begin{pmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ -3 & 8 & 0 \end{pmatrix}$$

3. Let A be a matrix of size $n \times n$ with entries in $\mathbb{R}$. Explain why each of the following statements holds.
   a. If $Ax = 0$ has only trivial solution, then A is row equivalent to the $n \times n$ identity matrix.
   b. If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$.
   c. If A is invertible, then the columns of A^{-1} is linearly independent

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ define by $T(x_1, x_2) = (6x_1 - 8x_2, -6x_1 + 9x_2)$. Show that T is invertible and find the determinant of the standard matrix of T.

3/a. $Ax = 0$ only has trivial soln.

A row equivalent to $n \times n$ identity?

row reduced echelon to A

exactly "n" pivots? (if there is a "free" variable, then infinite solution)

The entries, "the pivots" are "1".

So A must be I.

b/ $AD = I \Rightarrow$ must be "C" st $CA = I$

$A^{-1}AD = A^{-1}I$

$ID = A^{-1}I$

$\boxed{D = A^{-1}}$

$A(A^{-1}) = I$

$\underbrace{(A^{-1})}_{C} A = I$

choose $C = D = A^{-1}$

$\boxed{Ax = 0}$

A^{-1} is indep.

c/ ~~Aolumn~~ Column of A^{-1} indept if and only if $A^{-1}x = 0$ has only trivial soln. Multiply by A^2: $A(AA^{-1})x = 0$

$A \cdot Ix = 0$

Math 31 - Fall 2021 - Discussion 11

1. Find the inverse of the matrix
$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}.$$
You should get
$$A^{-1} = \begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{pmatrix}.$$
Use this to solve the system of equations $Ax = b$ where
$$b = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

2. Which of the following matrices are invertible? Explain your answers.
$$A = \begin{pmatrix} 16 & -4 \\ -12 & 3 \end{pmatrix}$$
$$B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$
$$C = \begin{pmatrix} 3 & 0 & -5 \\ 2 & 0 & 4 \\ -4 & 0 & 8 \end{pmatrix}$$
$$D = \begin{pmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ -3 & 8 & 0 \end{pmatrix}$$

3. Let A be a matrix of size $n \times n$ with entries in $\mathbb{R}$. Explain why each of the following statements holds.
   a. If $Ax = 0$ has only trivial solution, then A is row equivalent to the $n \times n$ identity matrix.
   b. If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$.
   c. If A is invertible, then the columns of A^{-1} is linearly independent

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ define by $T(x_1, x_2) = (6x_1 - 8x_2, -6x_1 + 9x_2)$. Show that T is invertible and find the determinant of the standard matrix of T.

$$T: \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto \begin{pmatrix} 6x_1 - 8x_2 \\ -6x_1 + 9x_2 \end{pmatrix}$$

Find A: (matrix represent T):

$$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6\cdot 0 - 8\cdot 1 \\ -6\cdot 0 + 9\cdot 1 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix}$$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6\cdot 1 - 8\cdot 0 \\ -6\cdot 1 + 9\cdot 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$$

$$A = \left[T\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \begin{bmatrix} 6 & -8 \\ -6 & 9 \end{bmatrix}$$

$$\det A = 6\cdot 9 - (-6)(-8) = 72 + 48$$
$$= 120.$$

$$A^{-1} = \frac{1}{120} \begin{bmatrix} 9 & 8 \\ 6 & 6 \end{bmatrix}.$$