Math 31 - Fall 2021 - Discussion 12

Topic: Sections 2.5 and 4.1.

1. Solve the equation $Ax = b$ by using the LU-factorization of A.

$$A = \begin{pmatrix} -2 & 4 & 5 \\ -4 & 9 & 9 \\ 10 & -17 & -25 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} -2 & 4 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & +3 \end{pmatrix}}_{U}, \quad b = \begin{pmatrix} 21 \\ 41 \\ -99 \end{pmatrix}$$

2. Find an LU-factorization of the matrix A.

$$A = \begin{pmatrix} 9 & 1 \\ 12 & 5 \end{pmatrix}$$

3. Let V be the set of vectors shown below.

$$V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x > 0, y \geq 0 \right\}$$

(3a) If u, v are in V, is $u + v$ in V?

(3b) Find u in V and c in $\mathbb{R}$ so that cu is not in V.

(3c) Is V a subspace of $\mathbb{R}^2$?

4. Consider the set of polynomials $p(t)$ of degree up to three, which satisfy $p(0) = 0$. Is this a subspace of $\mathcal{P}_3$?

5. Consider the following set of vectors.

$$V = \left\{ \begin{pmatrix} a + 2 \\ 2a + b \\ 3b - a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

(5a) Show that V a subspace of $\mathbb{R}^3$.

(5b) Find a finite set of vectors that spans V.

// $Ly = b$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 21 \\ \boxed{2} & 1 & 0 & 41 \\ \boxed{-5} & 3 & 1 & -99 \end{array}\right) \xrightarrow[\substack{-2R_1 + R_2 \to R_2 \\ 5R_1 + R_3 \to R_3}]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & -1 \\ 0 & \boxed{3} & 1 & 6 \end{array}\right)$$

$$\xrightarrow[\substack{-3R_2 + R_3 \\ \to R_3}]{} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 9 \end{array}\right) \qquad y = \begin{pmatrix} 21 \\ -1 \\ 9 \end{pmatrix}$$

$Ux = y$

$$\left(\begin{array}{ccc|c} -2 & 4 & 5 & 21 \\ 0 & 1 & \boxed{-1} & -1 \\ 0 & 0 & +3 & 9 \end{array}\right) \longrightarrow \left(\begin{array}{cccc} -2 & 4 & 5 & 21 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\xrightarrow[\substack{R_2 + R_3 \\ \to R_2}]{} \left(\begin{array}{ccc|c} -2 & 4 & \boxed{5} & 21 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow[\substack{-5R_3 + R_1 \\ \to R_1}]{} \left(\begin{array}{ccc|c} -2 & \boxed{4} & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\xrightarrow[\substack{-4R_2 + R_1 \\ \to R_1}]{} \left(\begin{array}{ccc|c} -2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Math 31 - Fall 2021 - Discussion 12

Topic: Sections 2.5 and 4.1.

1. Solve the equation $Ax = b$ by using the LU-factorization of A.

$$A = \begin{pmatrix} -2 & 4 & 5 \\ -4 & 9 & 9 \\ 10 & -17 & -25 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 21 \\ 41 \\ -99 \end{pmatrix}$$

2. Find an LU-factorization of the matrix A.

$$A = \begin{pmatrix} \boxed{9} & 1 \\ 12 & 5 \end{pmatrix}$$

3. Let V be the set of vectors shown below.

$$V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x > 0, y \geq 0 \right\}$$

   (3a) If u, v are in V, is $u + v$ in V?

   (3b) Find u in V and c in $\mathbb{R}$ so that cu is not in V.

   (3c) Is V a subspace of $\mathbb{R}^2$?

4. Consider the set of polynomials $p(t)$ of degree up to three, which satisfy $p(0) = 0$. Is this a subspace of $\mathcal{P}_3$?

5. Consider the following set of vectors.

$$V = \left\{ \begin{pmatrix} a + 2 \\ 2a + b \\ 3b - a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

   (5a) Show that V a subspace of $\mathbb{R}^3$.

   (5b) Find a finite set of vectors that spans V.

2)

$$L = \begin{pmatrix} \boxed{1} & 0 \\ \frac{4}{3} & 1 \end{pmatrix}$$

divide by 9: $? = \frac{12}{9} = \frac{4}{3}$

$$U = \begin{pmatrix} 9 & 1 \\ 0 & \frac{11}{3} \end{pmatrix}$$

$-\frac{12}{9} \cdot R_1 + R_2$

$-\frac{12}{9} \cdot 9 + 12 = 0$

$-\frac{12}{9} \cdot 1 + 5 =$

$-\frac{12}{9} + \frac{45}{9}$

$\frac{33}{9} = \frac{11}{3}$

Math 31 - Fall 2021 - Discussion 12

Topic: Sections 2.5 and 4.1.

1. Solve the equation $Ax = b$ by using the LU-factorization of A.

$$A = \begin{pmatrix} -2 & 4 & 5 \\ -4 & 9 & 9 \\ 10 & -17 & -25 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 21 \\ 41 \\ -99 \end{pmatrix}$$

2. Find an LU-factorization of the matrix A.

$$A = \begin{pmatrix} 9 & 1 \\ 12 & 5 \end{pmatrix}$$

3. Let V be the set of vectors shown below.

$$V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : \underline{x > 0}, \underline{y \geq 0} \right\}$$

(3a) If u, v are in V, is $u + v$ in V?

(3b) Find u in V and $\underline{c \text{ in } \mathbb{R}}$ so that cu is not in V.

(3c) Is V a subspace of $\mathbb{R}^2$?

4. Consider the set of polynomials $p(t)$ of degree up to three, which satisfy $p(0) = 0$. Is this a subspace of $\mathcal{P}_3$?

5. Consider the following set of vectors.

$$V = \left\{ \begin{pmatrix} a + 2 \\ 2a + b \\ 3b - a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

(5a) Show that V a subspace of $\mathbb{R}^3$.

(5b) Find a finite set of vectors that spans V.

3/a/ $u = \begin{pmatrix} a \\ b \end{pmatrix}$ with $a > 0$, $\boxed{b \geq 0}$

$v = \begin{pmatrix} c \\ d \end{pmatrix}$ with $c > 0$, $\boxed{d \geq 0}$

$u + v = \begin{pmatrix} \boxed{a+c} \\ \boxed{b+d} \end{pmatrix}$ $\boxed{a + c > 0}$?

Yes

$b + d \geq 0$?

Yes.

So $u + v$ is in V.

b/ $c = -2$, $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is in V

$\boxed{cu} = -2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ NOT in V.

c/ subspace: 1/ $u + v$ stay in V ✓ by a.

2/ cu stay in V :(nope

NOT a subspace of $\mathbb{R}^2$.

Math 31 - Fall 2021 - Discussion 12

Topic: Sections 2.5 and 4.1.

1. Solve the equation $Ax = b$ by using the LU-factorization of A.

$$A = \begin{pmatrix} -2 & 4 & 5 \\ -4 & 9 & 9 \\ 10 & -17 & -25 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 21 \\ 41 \\ -99 \end{pmatrix}$$

2. Find an LU-factorization of the matrix A.

$$A = \begin{pmatrix} 9 & 1 \\ 12 & 5 \end{pmatrix}$$

3. Let V be the set of vectors shown below.

$$V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x > 0, y \geq 0 \right\}$$

(3a) If u, v are in V, is $u + v$ in V?

(3b) Find u in V and c in $\mathbb{R}$ so that cu is not in V.

(3c) Is V a subspace of $\mathbb{R}^2$? V

4. Consider the set of polynomials $p(t)$ of degree up to three, which satisfy $p(0) = 0$. Is this a subspace of $\mathcal{P}_3$?

5. Consider the following set of vectors.

$$V = \left\{ \begin{pmatrix} a + 2 \\ 2a + b \\ 3b - a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

(5a) Show that V a subspace of $\mathbb{R}^3$.

(5b) Find a finite set of vectors that spans V.

1/ $p_1(x) := a_3x^3 + a_2x^2 + a_1x$ is in V.

$$p_1(0) = a_0 = 0$$

$p_2(x) = b_3x^3 + b_2x^2 + b_1x$ is in V.

$$b_0 = p_2(0) = 0$$

$$p_1(x) + p_2(x) = (a_3+b_3)x^3 + (a_2+b_2)x^2 + (a_1+b_1)x$$

$(p_1+p_2)(0) = 0$ is in V. $\boxed{\checkmark}$

scalar → real numbers

multiply with the vector stays in the space.

2/ c is in $\mathbb{R}$: (cp_1) ?

$cp_1(x) = ca_3x^3 + ca_2x^2 + ca_1x$ is in V

$$(cp_1)(0) = 0$$

Therefore V is a subspace.

Math 31 - Fall 2021 - Discussion 12

Topic: Sections 2.5 and 4.1.

1. Solve the equation $Ax = b$ by using the LU-factorization of A.

$$A = \begin{pmatrix} -2 & 4 & 5 \\ -4 & 9 & 9 \\ 10 & -17 & -25 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -5 & 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & 4 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 21 \\ 41 \\ -99 \end{pmatrix}$$

2. Find an LU-factorization of the matrix A.

$$A = \begin{pmatrix} 9 & 1 \\ 12 & 5 \end{pmatrix}$$

3. Let V be the set of vectors shown below.

$$V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x > 0, y \geq 0 \right\}$$

(3a) If u, v are in V, is $u + v$ in V?

(3b) Find u in V and c in $\mathbb{R}$ so that cu is not in V.

(3c) Is V a subspace of $\mathbb{R}^2$?

4. Consider the set of polynomials $p(t)$ of degree up to three, which satisfy $p(0) = 0$. Is this a subspace of $\mathcal{P}_3$?

5. Consider the following set of vectors.

$$V = \left\{ \begin{pmatrix} a + 2b \\ 2a + b \\ 3b - a \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

(5a) Show that V a subspace of $\mathbb{R}^3$.

(5b) Find a finite set of vectors that spans V.

5. 1/ $v = \begin{pmatrix} a+2b \\ 2a+b \\ 3b-a \end{pmatrix}$, $u = \begin{pmatrix} c+2d \\ 2c+d \\ 3d-c \end{pmatrix}$ in V.

$$v+u = \begin{pmatrix} (a+c) & 2(b+d) \\ 2(a+c) & b+d \\ 3(b+d) & -(a+c) \end{pmatrix} = \begin{pmatrix} e+2f \\ 2e+f \\ 3f-a \end{pmatrix}$$

$\downarrow$

is also in V :)

$a+c$: name it e

$b+d$: name it f

2/ k be a real number:

$$kv = k\begin{pmatrix} a+2b \\ 2a+b \\ 3b-a \end{pmatrix} = \begin{pmatrix} ka+2kb \\ 2ka+kb \\ 3kb-ka \end{pmatrix}$$

:)

same form: is in V.

V is a vector space.

b/ $a\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + b\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = V$: $\left\{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}\right\}$

linear combo