Math 31 - Fall 2021 - Discussion 13

1. Determine if the following set of vectors is a basis for $\mathbb{R}^3$.

$$\left\{ \underset{v_1}{\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}}, \underset{v_3}{\begin{pmatrix} -12 \\ 5 \\ 4 \end{pmatrix}} \right\}$$

2. Determine if the following set of vectors is a basis for $\mathbb{R}^3$. Is it linearly independent? Does it span $\mathbb{R}^3$?

$$\left\{ \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -6 \\ 6 \end{pmatrix} \right\}$$

3. Determine if the vector is in the null space of the matrix, and if it is in the column space of the matrix.

$$A = \begin{pmatrix} -6 & -2 & -5 \\ 6 & 4 & 4 \\ 4 & 0 & 4 \end{pmatrix}, \qquad w = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

4. Determine if the given set W is a vector space.

$$\left\{ \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} : \begin{array}{c} 3p + q = -s \\ -2p = -s + r \end{array} \right\}$$

5. Find a basis for the null space of the following matrix.

$$\begin{pmatrix} 1 & 8 & -6 & -4 & 1 \\ 0 & 1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1/ 3 vectors, basis for $\mathbb{R}^3$: 3 pivots

Check: for independence -

If independence $\Rightarrow$ basis,

$$\begin{pmatrix} 3 & 3 & -12 \\ -1 & -4 & 5 \\ 1 & 3 & 4 \end{pmatrix} \underset{\substack{\text{divide} \\ R_1 \text{ by } 3}}{\sim} \begin{pmatrix} 1 & 1 & -4 \\ -1 & -4 & 5 \\ 1 & 3 & 4 \end{pmatrix}$$

$$\underset{\substack{R_1+R_2 \\ R_1+R_3 \\ \to R_3}}{\sim} \begin{pmatrix} 1 & 1 & -4 \\ 0 & -3 & 1 \\ 0 & -1 & 9 \end{pmatrix} \underset{\substack{3R_3 \\ -R_2 \to R_3}}{\sim} \begin{pmatrix} 1 & 1 & -4 \\ 0 & -3 & 1 \\ 0 & 0 & 26 \end{pmatrix}$$

3 pivots $\Rightarrow$ indep.

$\mathbb{R}^4$: need 4 pivots

$$\underbrace{\begin{pmatrix} 1 & 2 \\ 2 & 0 \\ 0 & -4 \end{pmatrix}}_{2 \text{ vectors}} \text{ in } \mathbb{R}^3 \qquad \mathbb{R}^2 \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$$

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2/ indep $\boxed{\text{AND}}$ span $\mathbb{R}^3$ $\Rightarrow$ basis

because $v_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ so v_1, v_2, v_3, v_4

is NOT indep $\Rightarrow$ not a basis

$$* \quad \underbrace{\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & b_1 \\ -4 & -4 & 0 & -6 & b_2 \\ 0 & 0 & 0 & 6 & b_3 \end{array}\right)}_{Ax = b}$$

$\stackrel{?}{=}$ always has solution

$6x_4 = b_3$

so $\boxed{x_4 = \frac{b_3}{6}}$ ✓

$\boxed{x_3 \text{ free}}$ ✓

$-4x_1 - 4x_2 - 6x_4 = b_2$

$$x_1 = \frac{b_2 + 4x_2 + 6x_4}{-4}$$

$$x_1 + 3x_2 = b_1 \Rightarrow x_1 = b_1 - 3x_2$$

$4R_1 + R_2 \to R_2$

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & b_1 \\ 0 & 8 & 0 & -6 & 4b_1 + b_2 \\ 0 & 0 & 0 & 6 & b_3 \end{array}\right)$$

$8x_2 - 6x_4 = 4b_1 + b_2$

$8x_2 = 6x_4 + 4b_1 + b_2$

$8x_2 = b_3 + 4b_1 + b_2$

$$\boxed{x_2 = \frac{b_3 + 4b_1 + b_2}{8}}$$

We can find all x_1, x_2, x_3, x_4 so $\Rightarrow$

consistent $\Rightarrow v_1, v_2, v_3, v_4$ span $\mathbb{R}^3$.

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3/ a/ Is w in null space of A:

$Aw \stackrel{?}{=} 0$: then it is null space.

w in <u>null</u> space

$$\begin{pmatrix} -6 & -2 & -5 \\ 6 & 4 & 4 \\ 4 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-6 \cdot 2 + -2 \cdot (-1) + -5 \cdot -2)$

$6 \cdot 2 + 4 \cdot -1 + 4 \cdot (-2)$

$4 \cdot 2 + 0 \cdot -1 + 4 \cdot -2$

b/ Check if w in column space of A:

$Ax = w$ has solution $\Rightarrow$ w is $\boxed{\text{in}}$ column space

$$\left(\begin{array}{ccc|c} -6 & -2 & -5 & 2 \\ 6 & 4 & 4 & -1 \\ 4 & 0 & 4 & -2 \end{array}\right) \sim \begin{pmatrix} -6 & -2 & -5 & 2 \\ 0 & 2 & -1 & 1 \\ 0 & -\frac{4}{3} & \frac{2}{3} & -\frac{2}{3} \end{pmatrix}$$

$\frac{2}{3}R_1 + R_3 \to R_3$

$-\frac{10}{3} + \frac{12}{3}$

$\boxed{x_1 = \frac{1}{2} + x_3}$

$\frac{4}{3} - \frac{6}{2}$

$$\sim \begin{pmatrix} 6 & -2 & -5 & 2 \\ 0 & 2 & -1 & 1 \\ 0 & 2 & -1 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} \boxed{6} & -2 & -5 & 2 \\ 0 & \boxed{2} & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

x_3 free

$2x_2 - x_3 = 1$ ✓

$x_2 = \frac{1 + x_3}{2}$

$6x_1 - 2x_2 - 5x_3 = 2$

$6x_1 = 2 + 5x_3 + 2x_2 = 3 + 6x_3$

$+ 1 + x_3$

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4/ $3p + q = -s$

$-2p = -s + r$

<u>Trick</u>: $\begin{cases} 3p + 1q + 0r + s = 0 \\ -2p + 0q - 1r + s = 0 \end{cases}$

$$\underbrace{\begin{bmatrix} 3 & 1 & 0 & 1 \\ -2 & 0 & -1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \cdot \underbrace{\begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix}}_{\vec{w}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So $A\vec{w} = 0 \Rightarrow \vec{w}$ is in <u>Null space</u> $\boxed{\text{of } A}$,

W is the nullpace of A.

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basis

indep ✓

span : ✗

$\mathbb{R}^3$: 3 vectors

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null space

$\boxed{Ax = 0}$

5. Find a basis for the null space of the following matrix.

$$\underbrace{\begin{pmatrix} 1 & 8 & -6 & -4 & 1 \\ 0 & 1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{A}$$

5/

$$\left(\begin{array}{ccccc|c} \boxed{1} & 8 & -6 & -4 & 1 & 0 \\ 0 & \boxed{1} & -7 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 + 8x_2 - 6x_3 - 4x_4 + x_5 = 0$

$x_2 - 7x_3 + x_4 = 0$

$x_2 = 7x_3 - x_4$ ✓

$x_3 \quad x_4 \quad x_5$: free

$$x_1 = \underbrace{-8x_2}_{} + 6x_3 + 4x_4 - x_5$$

$$= -8(7x_3 - x_4) + 6x_3 + 4x_4 - x_5$$

$$x_1 = -50x_3 + 12x_4 - x_5$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -50x_3 + 12x_4 - x_5 \\ 7x_3 - x_4 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$$= x_3 \overset{v_1}{\begin{bmatrix} -50 \\ 7 \\ 1 \\ 0 \\ 0 \end{bmatrix}} + x_4 \overset{v_2}{\begin{bmatrix} 12 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}} + x_5 \overset{v_3}{\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$

$\{v_1, v_2, v_3\}$ is a <u>basis</u> for null space of A.