Math 31 - Fall 2021 - Week 9

1. Compute the determinant using a cofactor expansion across the third column.

$$\begin{pmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 4 & -2 \end{pmatrix}$$

$(-1)^{i+j}$

$a_{12}: (-1)^{1+2} = (-1)^3 = -1$

2. Which property of determinants is illustrated in this equation?

$$det\begin{pmatrix} 9 & 5 & 6 \\ 18 & -1 & 5 \\ -1 & 0 & 2 \end{pmatrix} = -det\begin{pmatrix} 18 & -1 & 5 \\ 9 & 5 & 6 \\ -1 & 0 & 2 \end{pmatrix}$$

3. Reduce the matrix, then use cofactor expansion to compute the determinant.

$$\begin{pmatrix} -1 & 4 & 9 & 0 \\ 3 & 4 & 5 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{pmatrix}$$

4. If

$$det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 5$$

then find the determinant of

$$\begin{pmatrix} d & e & f \\ 2a & 2b & 2c \\ g & h & i \end{pmatrix}$$

5. Is this matrix invertible?

$$\begin{pmatrix} 5 & -4 & 2 \\ 1 & 12 & 4 \\ 0 & -20 & -6 \end{pmatrix}$$

$$4\begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix} = 4\cdot(2\cdot 4 - 0\cdot 3)$$

$$-2\begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} \quad -2\,(3\cdot 4 - 0\cdot 0)$$

$$+(-2)\begin{vmatrix} 3 & 0 \\ 2 & 3 \end{vmatrix} \quad -2\,(3\cdot 3 - 2\cdot 0)$$

$$= 4\cdot 8 - 2\cdot 12$$

$$- 2\cdot 9 =$$

$$= 32 - 24 - 18 = -10$$

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$$\begin{pmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 4 & -2 \end{pmatrix}$$

2. Which property of determinants is illustrated in this equation?

$\boxed{\text{interchange row}}$

$$det\begin{pmatrix} 9 & 5 & 6 \\ 18 & -1 & 5 \\ -1 & 0 & 2 \end{pmatrix} = -det\begin{pmatrix} 18 & -1 & 5 \\ 9 & 5 & 6 \\ -1 & 0 & 2 \end{pmatrix}$$

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4. If

$$det\underbrace{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}}_{A} = 5$$

2 operations

(I) 1st interchange

then find the determinant of

$$\underbrace{\begin{pmatrix} d & e & f \\ 2a & 2b & 2c \\ g & h & i \end{pmatrix}}_{B} \longleftrightarrow \begin{pmatrix} d & e & f \\ a & b & c \\ g & h & i \end{pmatrix}$$

C : det C = -5

scale by 2

(II): 2nd

det B = 2(−5)
= −10.

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3 by 3: square matrix

$$\underbrace{\begin{pmatrix} 5 & -4 & 2 \\ 1 & 12 & 4 \\ 0 & -20 & -6 \end{pmatrix}}_{A}$$

$$\left(A \;\middle|\; \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \right)$$

$$\sim \left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \;\middle|\; \underbrace{\qquad\qquad}_{A^{-1}} \right)$$

Theorem 4, p. 173:

A square matrix is $\boxed{\text{INVERTIBLE}}$ if and only if $\det A \neq 0$.

$$\det \begin{vmatrix} 5 & -4 & 2 \\ 1 & 12 & 4 \\ 0 & -20 & -6 \end{vmatrix}$$

find the rows / column that has more ZERO.

$$5 \begin{vmatrix} 12 & 4 \\ -20 & -6 \end{vmatrix} - 1 \begin{vmatrix} -4 & 2 \\ -20 & -6 \end{vmatrix} + 0 \begin{vmatrix} & \\ & \end{vmatrix}$$

$$5 \cdot \left(-72 - (-80)\right) - 1\left(24 - (-40)\right) = 5 \cdot 8 - 64$$
$$= -24.$$

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$$\begin{pmatrix} 5 & -4 & 2 \\ 1 & 12 & 4 \\ 0 & -20 & -6 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 4 & 9 & 0 \\ \boxed{3} & 4 & 5 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{pmatrix} \xrightarrow[+R_2 \to R_2]{3R_1} \begin{pmatrix} -1 & 4 & 9 & 0 \\ 0 & 16 & 32 & 0 \\ \boxed{5} & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{pmatrix}$$

$$\xrightarrow[\to R_3]{5R_1+R_3} \begin{pmatrix} -1 & 4 & 9 & 0 \\ 0 & 16 & 32 & 0 \\ 0 & 24 & 51 & 6 \\ \boxed{4} & 2 & 4 & 3 \end{pmatrix} \xrightarrow[\to R_4]{4R_1+R_4} \begin{pmatrix} -1 & 4 & 9 & 0 \\ 0 & 16 & 32 & 0 \\ 0 & 24 & 51 & 6 \\ 0 & 18 & 40 & 3 \end{pmatrix}$$

$$\det A = (-1) \begin{vmatrix} 16 & 32 & 0 \\ 24 & 51 & 6 \\ 18 & 40 & 3 \end{vmatrix} = (-1) \cdot \frac{1}{16} \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 1 & 2 & 0 \\ 24 & 51 & 6 \\ 18 & 40 & 3 \end{vmatrix} \end{array} \begin{array}{c} \\ - \\ + \end{array}$$

$$= \frac{-1}{16} \left[0 \cdot \begin{vmatrix} & \end{vmatrix} - 6 \begin{vmatrix} 1 & 2 \\ 18 & 40 \end{vmatrix} + 3 \begin{vmatrix} 1 & 2 \\ 24 & 51 \end{vmatrix} \right]$$

$$= -\frac{1}{16} \left[-6 \cdot (40-36) + 3(51-48) \right]$$

$$= -\frac{1}{16} \left[-6 \cdot 4 + 3 \cdot 3 \right] = \frac{-1}{16} \cdot (-24+9)$$

$$= \frac{-1}{16}(-15) = \frac{15}{16}$$

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