Math 46 - Fall 2021 - Discussion section 002 and 005
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1 5.4 and 5.5: Method of Undetermined Coefficients

Exercise 1. Use the method of undetermined coefficients to solve the following differential equation:

$$y'' + y' = 4x$$

Answer: $y = 2x^2 - 4x + C_1 \times 1 + C_2 \times e^{-x}$

(y_p = $2x^2 - 4x$)

$4x$: they give power one : x degree 1

Step 1: start with homogeneous: $y'' + y' = 0$

$r^2 + r = 0 : r(r+1) = 0$

$r = 0$ or $r = -1$

$y = C_1 \cdot e^{0x} + C_2 e^{-1x}$

$= C_1 \cdot 1 + C_2 e^{-x}$

Step 2: find y_p : $y_p = x(Ax + B)$

they give • (polynomial) degree 1

"make $y_p = 5x^2$" : $y_p = x(Ax^2 + Bx + C)$

$y_p = x \cdot (Ax + B) = Ax^2 + Bx$: $y_p' = 2Ax + B$

$y_p'' = 2A$

$y'' + y' = 2x$:

$2A + (2Ax + B) = 4x$ so $2Ax = 4x \Rightarrow A = 2$

$2A + B = 0$ b/c no constant term on the right!

$B = -2A = -2 \cdot (2) = -4$

$y_p = 2x^2 - 4x$

Exercise 2. Use the method of undetermined coefficients to solve the following differential equation:

$$y'' + 6y' + 9y = 2e^{-x}$$

Answer: $y = \frac{1}{2}e^{-x} + C_1 \times e^{-3x} + C_2 \times xe^{-3x}$

Exercise 3. Use the method of undetermined coefficients to solve the following differential equation:

$$y'' + 4y = 4x$$

Answer: $y = x + C_1 \times \cos(2x) + C_2 \times \sin(2x)$

Exercise 4. Use the method of undetermined coefficients to solve the following differential equation:

$$y'' + 6y' + 9y = 2\sin(x)$$

Answer: $y = .16\sin(x) - .12\cos(x) + C_1 \times e^{-3x} + C_2 \times xe^{-3x}$

Exercise 5. Solve the following differential equation:

$$y'' + 4y = \sin^3 x$$

Hint: $\sin^3(x) = \frac{3\sin(x) - \sin(3x)}{4}$

Answer: $y = \frac{1}{4}\sin(x) + \frac{1}{20}\sin(3x) + C_1 \times \cos(2x) + C_2 \times \sin(2x)$

Step 1: Homogeneous: $y'' + 4y = 0$: $r^2 + 4 = 0$: $r = \pm 2i$

$r = \frac{0 \pm \sqrt{-16}}{2} = 0 \pm 2i$ $\qquad r = 0 \pm 2i$

$\alpha = 0$ $\qquad \beta = 2$

$y = C_1 e^0 \sin 2x + C_2 e^0 \cos 2x$

$$\boxed{y = C_1 \sin 2x + C_2 \cos 2x}$$