Khoi Vo **MATH_046_001_21F**

Assignment 6.1_6.2_APPLICATIONS_OF_LINEAR_2ND_ORDER_EQS due 12/01/2021 at 11:59pm PST

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2. (1 point)

A spring with a spring constant k of 20 pounds per foot is loaded with a 10-pound weight and allowed to reach equilibrium. It is then displaced 1 foot downward and released. If the weight experiences a retarding force in pounds equal to four times the velocity at every point, find the equation of motion.

$c = 4$ $k = 20$, $W = 10$ $W = mg$ $g = 32$

$m = \frac{W}{g} = \frac{10}{32} = \frac{5}{16}$

$y(t) =$ ______

where t is time (in seconds) and $y(t)$ is displacement (in feet).

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3. (1 point)

A spring with a spring constant k of 100 pounds per foot is loaded with 1-pound weight and brought to equilibrium. It is then stretched an additional 1 inch and released. Find the equation of motion, the amplitude, and the period. Neglect friction. Then

$y(t) =$ ______

where t is time in (seconds) and $y(t)$ is displacement (in feet).

Amplitude: ______ inch(es)

Period: ______ second(s).

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5. (1 point) A frictionless spring with a 5-kg mass can be held stretched 1.6 meters beyond its natural length by a force of 10 newtons. If the spring begins at its equilibrium position, but a push gives it an initial velocity of 1.5 m/sec, find the position of the mass after t seconds.

______ meters

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6. (1 point) A spring with a 3-kg mass and a damping constant 12 can be held stretched 1 meters beyond its natural length by a force of 4 newtons. Suppose the spring is stretched 2 meters beyond its natural length and then released with zero velocity.

In the notation of the text, what is the value $c^2 - 4mk$?

______ m^2kg^2/sec^2

Find the position of the mass, in meters, after t seconds. Your answer should be a function of the variable t of the form $c_1e^{\alpha t} + c_2e^{\beta t}$ where

$\alpha =$ ______ (the larger of the two)

$\beta =$ ______ (the smaller of the two)

$c_1 =$ ______ $c_2 =$ ______

2/ $my'' + cy' + ky = 0$

$$\frac{5}{16}y'' + 4y' + 20y = 0$$

$$\frac{5}{16}r^2 + 4r + 20 = 0$$

$$5r^2 + 64r + 320 = 0$$

$$r = \frac{-64 \pm \sqrt{64^2 - 20\cdot 320}}{10}$$

$$= \frac{-32}{10} \pm i\left(\frac{24}{5}\right)$$

$$y = e^{-\frac{32}{10}t}\left[c_1 \cos\frac{24}{5}t + c_2 \sin\frac{24}{5}t\right]$$

$y(0) = 1$: $\quad 1 = c_1 + c_2 \cdot 0 \Rightarrow c_1 = 1.$

$$y = e^{-\frac{32}{10}t}\left[\cos\frac{24}{5}t + c_2 \sin\frac{24}{5}t\right]$$

$y'(0) = 0$:

$$y' = -\frac{32}{10}e^{-\frac{32}{10}t}\cos\frac{24}{5}t - \frac{24}{5}e^{-\frac{32}{10}t}\sin\frac{24}{5}t - \frac{32}{10}e^{-\frac{32}{10}t}\cdot\frac{24}{5}t + c_2\frac{24}{5}e^{-\frac{32}{10}t}\cos\frac{24}{5}t$$

$$0 = \frac{-32}{5} - \frac{32}{5} + c_2\frac{24}{5}$$

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Assignment 9.2_HIGHER_ORDER_CONSTANT_COEFF_HOMOGENEOUS_EQ due 12/06/2021 at 11:59pm PST

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1. (1 point) Match the third order linear equations with their fundamental solution sets.

___1. $y''' - 5y'' + 6y' = 0$
___2. $ty''' - y'' = 0$
___3. $y''' + y' = 0$
___4. $y''' - 3y'' + y' - 3y = 0$
___5. $y''' - y'' - y' + y = 0$
___6. $y''' + 3y'' + 3y' + y = 0$

A. $\{e^{3t},\ \cos(t),\ \sin(t)\}$
B. $\{1,\ e^{3t},\ e^{2t}\}$
C. $\{1,\ t,\ t^3\}$
D. $\{1,\ \cos(t),\ \sin(t)\}$
E. $\{e^{-t},\ te^{-t},\ t^2e^{-t}\}$
F. $\{e^t,\ te^t,\ e^{-t}\}$

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1. (1 point)
Find y as a function of x if

$$y''' - 16y'' + 63y' = 240e^x,$$

$y(0) = 17, \;\; y'(0) = 28, \;\; y''(0) = 15.$
$y(x) =$ ______________