

2. (1 point)

A spring with a spring constant k of 20 pounds per foot is loaded with a 10-pound weight and allowed to reach equilibrium. It is then displaced 1 foot downward and released. If the weight experiences a retarding force in pounds equal to four times the velocity at every point, find the equation of motion.

$c = 4$ $m = \frac{W}{g} = \frac{10}{32} = \frac{5}{16}$

$y(t) =$ _____

where t is time (in seconds) and $y(t)$ is displacement (in feet).

3. (1 point)

A spring with a spring constant k of 100 pounds per foot is loaded with 1-pound weight and brought to equilibrium. It is then stretched an additional 1 inch and released. Find the equation of motion, the amplitude, and the period. Neglect friction. Then

$y(t) =$ _____

where t is time (in seconds) and $y(t)$ is displacement (in feet).

Amplitude: _____ inch(es)

Period: _____ second(s).

5. (1 point) A frictionless spring with a 5-kg mass can be held stretched 1.6 meters beyond its natural length by a force of 10 newtons. If the spring begins at its equilibrium position, but a push gives it an initial velocity of 1.5 m/sec, find the position of the mass after t seconds.

_____ meters

6. (1 point) A spring with a 3-kg mass and a damping constant 12 can be held stretched 1 meters beyond its natural length by a force of 4 newtons. Suppose the spring is stretched 2 meters beyond its natural length and then released with zero velocity.

In the notation of the text, what is the value $c^2 - 4mk$? _____ m^2kg^2/sec^2

Find the position of the mass, in meters, after t seconds. Your answer should be a function of the variable t of the form $c_1e^{\alpha t} + c_2e^{\beta t}$ where

$\alpha =$ _____ (the larger of the two)

$\beta =$ _____ (the smaller of the two)

$c_1 =$ _____ $c_2 =$ _____

$$y' = \frac{-32}{5} e^{-\frac{32}{5}t} \cos \frac{24t}{5} - e^{-\frac{32}{5}t} \cdot \left(\frac{24}{5}\right) \sin\left(\frac{24}{5}t\right)$$

$$-\frac{32}{5} e^{-\frac{32}{5}t} \cdot c_2 \sin \frac{24}{5}t + e^{-\frac{32}{5}t} c_2 \cdot \frac{24}{5} \cos \frac{24}{5}t$$

$$0 = -\frac{32}{5} + \frac{24}{5} c_2 \Rightarrow \boxed{c_2 = \frac{4}{3}}$$

$$2/ \underline{m}y'' + cy' + \underline{k}y = 0$$

$$\frac{5}{16}y'' + 4y' + 20y = 0$$

$$\frac{5}{16}r^2 + 4r + 20 = 0$$

$$\underline{5r^2 + 64r + 320 = 0}$$

$$r = \frac{-64 \pm \sqrt{64^2 - 4 \cdot 5 \cdot 320}}{10}$$

$$= \frac{-64}{10} \pm i \frac{24}{5} = \frac{-32}{5} \pm i \frac{24}{5}$$

$$y = e^{-\frac{32}{5}t} \left[\cancel{c_1} \cos \frac{24}{5}t + c_2 \sin \frac{24}{5}t \right]$$

$$y(0) = 1 : 1 = c_1 + c_2 \cdot 0 \Rightarrow c_1 = 1$$

$$y'(0) = 0 :$$

2. (1 point)

A spring with a spring constant k of 20 pounds per foot is loaded with a 10-pound weight and allowed to reach equilibrium. It is then displaced 1 foot downward and released. If the weight experiences a retarding force in pounds equal to four times the velocity at every point, find the equation of motion.

$$y(t) = \underline{\hspace{2cm}}$$

where t is time (in seconds) and $y(t)$ is displacement (in feet).

3. (1 point)

A spring with a spring constant k of 100 pounds per foot is loaded with 1-pound weight and brought to equilibrium. It is then stretched an additional 1 inch and released. Find the equation of motion, the amplitude, and the period. Neglect friction. Then

$$y(t) = \underline{\hspace{2cm}}$$

where t is time in (seconds) and $y(t)$ is displacement (in feet).

Amplitude: _____ inch(es)

Period: _____ second(s).

5. (1 point) A frictionless spring with a 5-kg mass can be held stretched 1.6 meters beyond its natural length by a force of 10 newtons. If the spring begins at its equilibrium position, but a push gives it an initial velocity of 1.5 m/sec, find the position of the mass after t seconds.

6. (1 point) A spring with a 3-kg mass and a damping constant 12 can be held stretched 1 meters beyond its natural length by a force of 4 newtons. Suppose the spring is stretched 2 meters beyond its natural length and then released with zero velocity.

In the notation of the text, what is the value $c^2 - 4mk$? _____ $\text{m}^2\text{kg}^2/\text{sec}^2$

Find the position of the mass, in meters, after t seconds. Your answer should be a function of the variable t of the form $c_1 e^{\alpha t} + c_2 e^{\beta t}$ where

$\alpha =$ _____ (the larger of the two)

$\beta =$ _____ (the smaller of the two)

$c_1 =$ _____ $c_2 =$ _____

$$y'(0) = 0: \quad c_1 \left(-2 - \frac{2}{3}\sqrt{6}\right) e^{\left(-2 - \frac{2}{3}\sqrt{6}\right)t} + c_2 \left(-2 + \frac{2}{3}\sqrt{6}\right) e^{\left(-2 + \frac{2}{3}\sqrt{6}\right)t}$$

$$0 = c_1 \left(-2 - \frac{2}{3}\sqrt{6}\right) + c_2 \left(-2 + \frac{2}{3}\sqrt{6}\right)$$

$$0 = (2 - c_2) \left(-2 - \frac{2}{3}\sqrt{6}\right) + c_2 \left(-2 + \frac{2}{3}\sqrt{6}\right)$$

$$0 = -4 - \frac{4}{3}\sqrt{6} + 2c_2 + \frac{2}{3}\sqrt{6}c_2 - 2c_2 + \frac{2}{3}\sqrt{6}c_2$$

$$m = 3$$

$$c = 12$$

$$F = kx$$

$$4 = k \cdot 1 \Rightarrow k = 4$$

$$c^2 - 4mk$$

$$= 12^2 - 4 \cdot 3 \cdot 4$$

$$= 12 \cdot 8 = \underline{\underline{96}}$$

$$my'' + cy' + ky = 0$$

$$3y'' + 12y' + 4y = 0$$

$$3r^2 + 12r + 4 = 0$$

$$r = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 3 \cdot 4}}{6}$$

$$= \frac{-12 \pm \sqrt{96}}{6} = \frac{-12 \pm 4\sqrt{6}}{6}$$

$$= -2 \pm \frac{2}{3}\sqrt{6}$$

$$y = c_1 e^{(-2 - \frac{2}{3}\sqrt{6})t} + c_2 e^{(-2 + \frac{2}{3}\sqrt{6})t}$$

$$y(0) = 2 : c_1 + c_2 = 2$$

1. (1 point) Match the third order linear equations with their fundamental solution sets.

—1. $y''' - 5y'' + 6y' = 0$

—2. $ty''' - y'' = 0$

—3. $y''' + y' = 0$

—4. $y''' - 3y'' + y' - 3y = 0$

—5. $y''' - y'' - y' + y = 0$

—6. $y''' + 3y'' + 3y' + y = 0$

A. $\{e^{3t}, \cos(t), \sin(t)\}$

B. $\{1, e^{3t}, e^{2t}\}$

C. $\{1, t, t^3\}$

D. $\{1, \cos(t), \sin(t)\}$

E. $\{e^{-t}, te^{-t}, t^2e^{-t}\}$

F. $\{e^t, te^t, e^{-t}\}$

1. (1 point)

Find y as a function of x if

$$y''' - 16y'' + 63y' = 240e^x,$$

$$y(0) = 17, \quad y'(0) = 28, \quad y''(0) = 15.$$

$$y(x) = \underline{\hspace{2cm}}$$