Applied Linear Algebra - MATH 31
UC Riverside - Fall 2021 - Practice Final Exam

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(1) Let

$$A = \begin{pmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{pmatrix}, \qquad b = \begin{pmatrix} 4 \\ 7 \\ -6 \end{pmatrix}.$$

(1a) Write the reduced row echelon form of A.

(1b) Write the general solution for the system $Ax = b$.

(1c) Write the general solution for the system $Ax = 0$.

(2) Consider the set of vectors

$$v_1 = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ -9 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 2 \\ -6 \end{pmatrix}$$

and let

$$b = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$$

.

(2a) Are $\{v_1, v_2, v_3\}$ linearly independent?

(2b) Is b in the span of $\{v_1, v_2, v_3\}$?

(3) Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotating vectors 90° counter clockwise about the origin. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear map $T(a, b, c) = (2a - c, a + b + c)$.

(3a) Find the matrix A_R associated with the transformation R.

(3b) Find the matrix A_T associated with the linear transformation R.

(3c) Find the matrix $A_{R \circ T}$ associated with the linear transformation given by the composition $R \circ T$.

(4) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 0 & -5 & 3 \\ -1 & -6 & 1 \end{pmatrix}.$$

1/ $[A \mid b]$

$$= \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{array}\right] \underset{\substack{R_2 - R_1 \\ \to R_2}}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ \underline{0} & \underline{5} & \underline{-15} & \underline{15} \end{array}\right]$$

$$\underline{3}R_2 + R_3 \to R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \underset{\substack{-3R_2 \\ +R_1 \\ \to R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

1a/ is $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

1/b: x_3 free

$x_2 - 3x_3 = 3 \Rightarrow x_2 = 3x_3 + 3$

$x_1 + 4x_3 = -5 \Rightarrow x_1 = -4x_3 - 5$

$$\vec{x} = \begin{bmatrix} -4x_3 - 5 \\ 3x_3 + 3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}, \ x_3 \in \mathbb{R}.$$

1c/ x_3 free, $x_2 - 3x_3 = 0 \Rightarrow x_2 = 3x_3$; $x_1 + 4x_3 = 0$

$x_1 = -4x_3$

$x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$ with $x_3 \in \mathbb{R}$.

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$[v_1 \ v_2 \ v_3] \sim$? solution non-zero

(2a) Are $\{v_1, v_2, v_3\}$ linearly independent?

(2b) Is b in the span of $\{v_1, v_2, v_3\}$?

$\underline{[v_1 \ v_2 \ v_3 | b]} \underset{\text{row reduced}}{\sim}$ find solution

(3) Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotating vectors $90°$ counter clockwise about the origin. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear map $T(a, b, c) = (2a - c, a + b + c)$.

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2/ $[\, v_1 \;\; v_2 \;\; v_3 \mid b \,]$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & -3 & -6 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & \underline{1} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

2a/ $[\, v_1 \;\; v_2 \;\; v_3 \,] \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \boxed{0} \end{bmatrix}$ third column has <u>NO</u> pivot

<u>not</u> lin indep

2b/ $[\, v_1 \;\; v_2 \;\; v_3 \mid b \,] \sim \left[\begin{array}{cccc} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \boxed{0} & \underline{0} \end{array}\right]$ is

x_3 free

consistent so : b is in the span $\{v_1, v_2, v_3\}$

$x_2 + 2x_3 = 1 \Rightarrow x_2 = -2x_3 + 1$

$x_1 + 3x_2 + x_3 = 1 \Rightarrow x_1 = -3x_2 - x_3 + 1$

$= -3(-2x_3 + 1) - x_3 + 1$

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$2a - c$
$2 \cdot 0 - 1$

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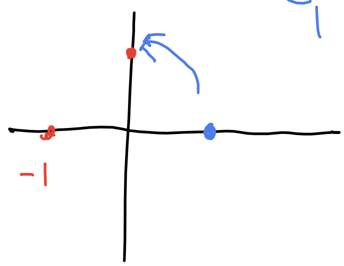
3/ A_R : $R\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$

$e_1 = \begin{pmatrix}1\\0\end{pmatrix}$

3a/

$R\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$



$A_R = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

$2a-c = 2\cdot 0-1 = -1$

$2a-c = 2\cdot 0 - 0 = 0$

3b/ $T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$ ✓, $T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$, $T=\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

$a=0,\ b=0,\ c=1$

$a+b+c$

$a=0,\ b=1,\ c=0$

$a+b+c = 1$

$A_T = \begin{bmatrix}2 & 0 & -1\\1 & 1 & 1\end{bmatrix}$

3c/ $A_{R\circ T}$ = multiply : RT

$$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}2 & 0 & -1\\1 & 1 & 1\end{pmatrix} = \begin{pmatrix}-1 & -1 & -1\\2 & 0 & -1\end{pmatrix}$$

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4/ $[A \mid I]$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 3 & 0 & 1 & 0 \\ -1 & -6 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{R_3+R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 3 & 0 & 1 & 0 \\ 0 & -2 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\underset{5R_3 - 2R_2 \to R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 3 & 0 & 1 & 0 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right]$$

$$\underset{3R_3 + R_2 \to R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 0 & 15 & -5 & 15 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right] \xrightarrow{\frac{R_2}{-5}} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & -3 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right]$$

$$\underset{-4R_2 + R_1 \to R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 13 & -4 & 12 \\ 0 & 1 & 0 & -3 & 1 & -3 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right]$$

$$\underset{\frac{R_3}{-1}}{\sim} \quad A^{-1} = \begin{bmatrix} 13 & -4 & 12 \\ -3 & 1 & -3 \\ -5 & 2 & -5 \end{bmatrix}$$

(5) Let H be the subspace of $\mathbb{R}^4$ given by

$$H = \left\{ \begin{pmatrix} 2a - 4b + 5c \\ 3a - d \\ b - 3c + d \\ 3c \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

(5a) Find a basis for H.

(5b) Find a the dimension of H.

(6) Let

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$

(6a) Find a basis for the row space of A.

(6b) Find a basis for the column space of A.

(6c) Find a basis for the null space of A.

(7) Let

$$A = \begin{pmatrix} 4 & -9 & 5 \\ 11 & 2 & 0 \\ 6 & -2 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \qquad C = \begin{pmatrix} d & e & f \\ 7a & 7b & 7c \\ g + 3d & h + 3e & i + 3f \end{pmatrix}$$

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(8) Let

$$A = \begin{pmatrix} 4 & 6 \\ 6 & -12 \end{pmatrix}$$

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5/

$$H = a\underbrace{\begin{pmatrix}2\\3\\0\\0\end{pmatrix}}_{v_1} + b\underbrace{\begin{pmatrix}-4\\0\\1\\0\end{pmatrix}}_{v_2} + c\underbrace{\begin{pmatrix}5\\0\\-3\\3\end{pmatrix}}_{v_3} + d\underbrace{\begin{pmatrix}0\\-1\\1\\0\end{pmatrix}}_{v_4}$$

1/ v_1, v_2, v_3, v_4 $\boxed{\text{spans}}$ H: ✓

$av_1 + bv_2 + cv_3 + dv_4 = H$

2/ check for lin indep?

$$\begin{pmatrix}2 & -4 & 5 & 0\\ \boxed{3} & 0 & 0 & -1\\ 0 & 1 & -3 & 1\\ 0 & 0 & 3 & 0\end{pmatrix} \underset{\substack{-3R_1 \\ +R_2 \to R_2}}{\overset{R_1/2}{\sim}} \begin{pmatrix}1 & -2 & \frac{5}{2} & 0\\ 0 & 6 & -15/2 & -1\\ 0 & 1 & -3 & 1\\ 0 & 0 & 3 & 0\end{pmatrix} \begin{matrix}-6R_3\\ +R_2 \to R_3\end{matrix}$$

$$\begin{pmatrix}1 & -2 & \frac{5}{2} & 0\\ 0 & 6 & -\frac{15}{2} & -1\\ 0 & 0 & \frac{21}{2} & -7\\ 0 & 0 & 3 & 0\end{pmatrix} \underset{\to R_4}{\overset{2R_3 - 7R_4}{\sim}} \begin{pmatrix}\boxed{1} & -2 & \frac{5}{2} & 0\\ 0 & \boxed{6} & -\frac{15}{2} & -1\\ 0 & 0 & \boxed{\frac{21}{2}} & -7\\ 0 & 0 & 0 & \boxed{-14}\end{pmatrix}$$

4 pivots: $\{v_i\}$ are in $\mathbb{R}^4$ $\Rightarrow$ lin indep.

$\{v_1, v_2, v_3, v_4\}$ is a basis for H: (5a)

5b: dim = 4 b/c there are 4 vectors in the basis.

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$$A = \begin{pmatrix} 4 & -9 & 5 \\ 11 & 2 & 0 \\ 6 & -2 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \qquad C = \begin{pmatrix} d & e & f \\ 7a & 7b & 7c \\ g + 3d & h + 3e & i + 3f \end{pmatrix}$$

(7a) Find the determinant of A.

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(8) Let

$$A = \begin{pmatrix} 4 & 6 \\ 6 & -12 \end{pmatrix}$$

(8a) Find the eigenvalues of A.

(8b) For each eigenvalue of A find a basis of the corresponding space of eigenvectors.

6/ $A \sim \begin{pmatrix} 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \\ -3 & 6 & -1 & 1 & -7 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & -1 & -2 & 2 \\ 0 & 0 & 5 & 10 & -10 \end{pmatrix}$

$2R_1 - R_2 \to R_2$

$3R_1 + R_3 \to R_3$

$\sim \begin{pmatrix} \boxed{1} & -2 & 2 & 3 & -1 \\ 0 & 0 & \boxed{-1} & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \underset{\substack{2R_2 + R_1 \\ \to R_1}}{\sim} \begin{pmatrix} \boxed{1} & -2 & 0 & -1 & 3 \\ 0 & 0 & \boxed{+1} & +2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$x_3 \quad x_4 \quad x_5$

6a/ row of the non-zero (pivots): $(1 \ -2 \ 0 \ -1 \ 3)$, $(0 \ 0 \ +1 \ +2 \ -2)$

6b/ pivot is in 1^{st} and 3^{rd} column : basis: $\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$

in the original A

Do NOT take the row-reduced.

6c/ $x_1 - 2x_2 - x_4 - 3x_5 = 0$

$x_3 + 2x_4 - 2x_5 = 0$

$\begin{pmatrix} 2x_2 + x_4 + 3x_5 \\ x_2 \\ -2x_4 + 2x_5 \\ x_4 \\ x_5 \end{pmatrix}$

$x_2 \underbrace{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}_{v_1} + x_4 \underbrace{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}_{v_2} + x_5 \underbrace{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}_{v_3}$

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$$A = \begin{array}{cc} \begin{array}{ccc} + & - & + \end{array} & \\ \begin{pmatrix} 4 & -9 & 5 \\ 11 & 2 & 0 \\ 6 & -2 & 4 \end{pmatrix} & \begin{array}{c} \\ - \\ + \end{array} \end{array}, \qquad B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \qquad C = \begin{pmatrix} d & e & f \\ 7a & 7b & 7c \\ g + 3d & h + 3e & i + 3f \end{pmatrix}$$

det B = 10

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7/a/ $5\begin{vmatrix} 11 & 2 \\ 6 & -2 \end{vmatrix} - 0\begin{vmatrix} 4 & -9 \\ 6 & -2 \end{vmatrix} + 4\begin{vmatrix} 4 & -9 \\ 11 & 2 \end{vmatrix}$

$= 5\cdot(-22-12) - 0 + 4(8+99) = 258.$

b/ To get C, starting from B:

Swap r1 and r2, multiply r_1 by 7,

(r_3 is replaced by $\underbrace{r_3 + 3r_2}$ a linear combination of rows)

These actions will:

scale the determinant of B by: (-1), scale the det of B by 7,

Scale by 1

So $\det C = 10\cdot(-1)\cdot(7)\cdot(1) = \underline{-70}$

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$A = \begin{pmatrix} 4 & 6 \\ 6 & -12 \end{pmatrix}$ | a/ $\det[A - \lambda I]$

$= \det \begin{pmatrix} 4-\lambda & 6 \\ 6 & -12-\lambda \end{pmatrix} = (4-\lambda)(-12-\lambda) - 36$

$= \lambda^2 + 8\lambda - 84 = 0$

$\lambda_1 = 6$ and $\lambda_2 = -14$

* $\lambda_1 = 6$:

$A - 6I = \begin{pmatrix} 4-6 & 6 \\ 6 & -12-6 \end{pmatrix} = \begin{pmatrix} -2 & 6 \\ 6 & -18 \end{pmatrix} \sim \begin{pmatrix} -2 & 6 \\ 0 & 0 \end{pmatrix}$

$(A - 6I)x = 0$ $\qquad \sim \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix}$

$x_1 - 3x_2 = 0$: $x_1 = 3x_2$, x_2 free

$x_2 \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ so basis for space of eigenvectors is $v_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

$\lambda = -14$: $A + 14I = \begin{pmatrix} 4+14 & 6 \\ 6 & -12+14 \end{pmatrix} \sim \begin{pmatrix} 18 & 6 \\ 6 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1/3 \\ 0 & 0 \end{pmatrix}$

so $x_1 + \frac{1}{3}x_2 = 0 \Rightarrow \begin{pmatrix} -\frac{1}{3}x_2 \\ x_2 \end{pmatrix}$: $x_2 \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix}$: basis for space of eigenvectors is $v_2 = \begin{pmatrix} -1/3 \\ 1 \end{pmatrix}$.

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