Applied Linear Algebra - MATH 31
UC Riverside - Fall 2021 - Practice Final Exam

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(1) Let

$$A = \begin{pmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{pmatrix}, \qquad b = \begin{pmatrix} 4 \\ 7 \\ -6 \end{pmatrix}.$$

(1a) Write the reduced row echelon form of A.

(1b) Write the general solution for the system $Ax = b$.

(1c) Write the general solution for the system $Ax = 0$.

(2) Consider the set of vectors

$$v_1 = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ -9 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 2 \\ -6 \end{pmatrix}$$

and let

$$b = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$$

.

(2a) Are $\{v_1, v_2, v_3\}$ linearly independent?

(2b) Is b in the span of $\{v_1, v_2, v_3\}$?

(3) Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotating vectors 90° counter clockwise about the origin. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear map $T(a, b, c) = (2a - c, a + b + c)$.

(3a) Find the matrix A_R associated with the transformation R.

(3b) Find the matrix A_T associated with the linear transformation R.

(3c) Find the matrix $A_{R \circ T}$ associated with the linear transformation given by the composition $R \circ T$.

(4) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 0 & -5 & 3 \\ -1 & -6 & 1 \end{pmatrix}.$$

1/ $[A \mid b]$

$$= \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{array}\right] \underset{R_2 - R_1 \to R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & \underline{\underline{5}} & \underline{\underline{-15}} & \underline{\underline{15}} \end{array}\right]$$

$3R_2 + R_3 \to R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \underset{-3R_2 + R_1 \to R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

1a/ is $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

1/b: x_3 free

$x_2 - 3x_3 = 3 \Rightarrow x_2 = 3x_3 + 3$

$x_1 + 4x_3 = -5 \Rightarrow x_1 = -4x_3 - 5$

$$\vec{x} = \begin{bmatrix} -4x_3 - 5 \\ 3x_3 + 3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}, \; x_3 \in \mathbb{R}.$$

1c/ x_3 free, $x_2 - 3x_3 = 0 \Rightarrow x_2 = 3x_3$; $x_1 + 4x_3 = 0$

$x_1 = -4x_3$

$x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$ with $x_3 \in \mathbb{R}$.

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$[v_1 \; v_2 \; v_3] \sim$? solution ^ non-zero

(2a) Are $\{v_1, v_2, v_3\}$ linearly independent?

(2b) Is b in the span of $\{v_1, v_2, v_3\}$?

$[v_1 \; v_2 \; v_3 \,|\, b] \underset{\text{row reduced}}{\sim}$ find solution

(3) Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotating vectors 90° counter clockwise about the origin. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear map $T(a, b, c) = (2a - c, a + b + c)$.

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$2/ \; [\, v_1 \;\; v_2 \;\; v_3 \mid b \,]$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & -3 & -6 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & \underline{1} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$2a/ \; [\, v_1 \;\; v_2 \;\; v_3 \,] \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \boxed{0} \end{bmatrix}$ third column has <u>NO</u> pivot

<u>not</u> lin indep

$2b/ \; [\, v_1 \;\; v_2 \;\; v_3 \mid b \,] \sim \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \boxed{0} & 0 \end{bmatrix}$ is

x_3 free

consistent so : b is in the span $\{v_1, v_2, v_3\}$

$$x_2 + 2x_3 = 1 \Rightarrow x_2 = -2x_3 + 1$$

$$x_1 + 3x_2 + x_3 = 1 \Rightarrow x_1 = -3x_2 - x_3 + 1$$

$$= -3(-2x_3 + 1) - x_3 + 1$$

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$2a - c$
$2 \cdot 0 - 1$

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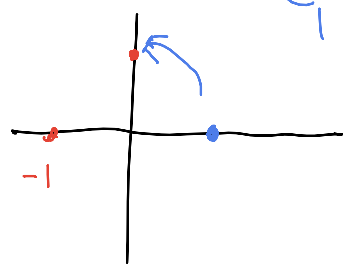
3/ A_R : $R\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$

3a/ $R\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$



$e_1 = \begin{pmatrix}1\\0\end{pmatrix}$

$A_R = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

$2a - c = 2\cdot 0 - 1 = -1$

$2a - c = 2\cdot 0 - 0$
$= 0$

3b/ $T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$ ✓ , $T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$, $T = \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

$a = 0, b = 0, c = 1$

$a + b + c$

$a + b + c$
$= 1$

$a = 0, b = 1, c = 0$

$A_T = \begin{bmatrix}2 & 0 & -1\\1 & 1 & 1\end{bmatrix}$

3c/ $A_{R\circ T}$ = multiply : RT

$$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}2 & 0 & -1\\1 & 1 & 1\end{pmatrix} = \begin{pmatrix}-1 & -1 & -1\\2 & 0 & -1\end{pmatrix}$$

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4/ [A | I]

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 3 & 0 & 1 & 0 \\ -1 & -6 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{R_3 + R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 3 & 0 & 1 & 0 \\ 0 & -2 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\underset{3R_3 + -2R_2 \to R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 3 & 0 & 1 & 0 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right]$$

$$\underset{3R_3 + R_2 \to R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & 0 & 15 & -5 & 15 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right] \xrightarrow{\frac{R_2}{-5}} \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & -3 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right]$$

$$\xrightarrow{-4R_2 + R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 13 & -4 & 12 \\ 0 & 1 & 0 & -3 & 1 & -3 \\ 0 & 0 & -1 & 5 & -2 & 5 \end{array}\right]$$

$$\underset{\frac{R_3}{-1}}{\sim} A^{-1} = \begin{bmatrix} 13 & -4 & 12 \\ -3 & 1 & -3 \\ -5 & 2 & -5 \end{bmatrix}$$

(5) Let H be the subspace of $\mathbb{R}^4$ given by

$$H = \left\{ \begin{pmatrix} 2a - 4b + 5c \\ 3a - d \\ b - 3c + d \\ 3c \end{pmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

(5a) Find a basis for H.

(5b) Find a the dimension of H.

(6) Let

$$A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$$

(6a) Find a basis for the row space of A.

(6b) Find a basis for the column space of A.

(6c) Find a basis for the null space of A.

(7) Let

$$A = \begin{pmatrix} 4 & -9 & 5 \\ 11 & 2 & 0 \\ 6 & -2 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \qquad C = \begin{pmatrix} d & e & f \\ 7a & 7b & 7c \\ g + 3d & h + 3e & i + 3f \end{pmatrix}$$

(7a) Find the determinant of A.

(7b) If the determinant of B is 10, find the determinant of C.

(8) Let

$$A = \begin{pmatrix} 4 & 6 \\ 6 & -12 \end{pmatrix}$$

(8a) Find the eigenvalues of A.

(8b) For each eigenvalue of A find a basis of the corresponding space of eigenvectors.