3/ $x\,\partial_x u + y\,\partial_y u = 2u$

$\boxed{u(x,y) = 1}$ on circle $= \{x^2 + y^2 = 1\}$

Step 1: $a = x$, $b = y$, $c = 2u$

(1) Step 2: parametrization

$\cos^2(s) + \sin^2(s) = 1$

$x_0(s) = \cos(s)$, $y_0(s) = \sin(s)$, $u_0(s) = 1$

(2) Step 3: $\partial_t x = x$, $\boxed{x_0(s) = \cos(s)}$: (1)

$t = 0 : \cos(s)$

$\boxed{\partial_t y = y, \quad y_0(s) = \sin(s)}$: (2)

$\partial_t u = 2u$, $u_0(s) = 1$: (3)

(1): $\underbrace{x(t,s)}_{f} = e^t \cdot K$: $K = \cos(s)$: $x(t,s) = e^t \cos(s)$

$f = ?$

$\boxed{\frac{df}{dt} = f}$

$\frac{df}{f} = dt \xrightarrow{\text{integrate}} \int \frac{df}{f} = \int dt$

$= \ln f = t + C$

$f = e^{t+c}$

$= e^t \cdot K$

(2): $\partial_t y = y \quad , \quad y_0(s) = \sin(s)$

$$y(t,s) = e^{t} \cdot \sin(s) \qquad (5)$$

(3) $\partial_t u = \underline{2}u \quad , \quad u_0(s) = 1$

$$u(t,s) = e^{2t} \cdot 1 = e^{2t} \qquad (6)$$

$$x(t,s) = e^{t} \cdot \cos(s) \qquad (5)$$

$$u = \underset{\text{something in } x}{\#x} + \underset{\text{something in } y}{\#y}$$

$$\begin{cases} x = \cos(s) \cdot e^{t} \\ y = \sin(s) \cdot e^{t} \end{cases} \quad \text{square} \to$$

$$x^2 = \cos^2(s) \cdot e^{2t} \quad , \quad y^2 = \sin^2(s) \cdot e^{2t}$$

$$x^2 + y^2 = e^{2t}(\cos^2 s + \sin^2 s) = e^{2t} = u$$

1/ $1 \cdot \partial_x u + \partial_y u = 1$, $u(x, 0) = \sin(x)$

Step 1: $a = 1$, $b = 1$, $c = 1$

Step 2: Parametrization

$x_0(s) = s$, $y_0(s) = 0$, $u_0(s) = \sin(s)$

Step 3:

$$\begin{cases} \partial_t x = 1, & x_0(s) = s \quad (1) \\ \partial_t y = 1, & y_0(s) = 0 \quad (2) \\ \partial_t u = 1, & u_0(s) = \sin(s) \quad (3) \end{cases}$$

$$\frac{df}{dt} = 1 \Rightarrow f = t + K$$

$$\begin{cases} x(t,s) = t + s & \longrightarrow x = t + s \\ y(t,s) = t + 0 = t & \longrightarrow y = t \\ u(t,s) = t + \sin(s) \end{cases}$$

$s = x - t = x - y$

$$\boxed{u(x,y) = y + \sin(x-y)}$$

$* \underline{\underline{2}} *$ $x\,\partial_x u + (x+y)\,\partial_y u = 1$, $\boxed{u(1,y) = y}$

Step 1: $a = x$, $b = x+y$, $c = 1$

Step 2: Parametrization :

$x_0(s) = 1$, $y_0(s) = s$, $u_0(s) = s$

Step 3: $\partial_t x = x$, $x(0,s) = 1$ (1)

$\partial_t y = x+y$, $y_0(s) = s$ (2)

$\partial_t u = 1$, $u_0(s) = s$ (3)

(1): $\boxed{x(t,s) = e^t}$ (4) $\quad \boxed{u = t+s}$ (6)

$\partial_t y = e^t + y \Rightarrow \partial_t - y = e^t$

$\partial_t(e^{-t} y) \underset{\text{product rule}}{=} e^{-t}(\underbrace{\partial_t y - y}_{e^t}) = e^{-t} \cdot e^t = \boxed{1}$

Integrate wrt $\cancel{s}$ on $[0,t]$:

$e^{-t} y(t,s) - e^0 y(0,s) = t$

$e^{-t} y(t,s) - s = t$

$\boxed{y(t,s) = (s+t)e^t}$ (5)

$\boxed{t = \ln x}$ (from 4)

$y = x(\ln x + s)$

$\Rightarrow \boxed{s = \frac{y}{x} - \ln x}$

$u = t+s$

$= \frac{y}{x}$.