$*\underline{4}*$ $\quad \partial_t u - \partial_x^3 u = x^2 - t + \cos(2x-t) \quad (*)$

$L(u) = \partial_t u - \partial_x^3 u$

① $L(u) = x^2$: find u_1

② $L(u) = -t$: find u_2

③ $L(u) = \cos(2x-t)$: find u_3

$u = u_1 + u_2 + u_3$ by superposition principle

①: $\partial_t u - \partial_x^3 u = x^2$

$u_1 = C_1 x^5 \implies -\partial_x^3 u_1 = -60 C_1 x^2 = x^2$

So $\boxed{u_1 = -\frac{1}{60} x^5}$ $\qquad C_1 = -\frac{1}{60}$

② $\partial_t u - \partial_x^3 u = -t$: guess $u_2 = C_2 t^2$

$\boxed{u_2 = -\frac{1}{2} t^2}$ $\qquad \partial_t u_2 - \underbrace{\partial_x^3 u}_{0} = 2C_2 t = -t$

$C_2 = -\frac{1}{2}$

③ $\partial_t u - \partial_x^3 u = \cos(2x-t)$

guess: $u_3 = c_3 \sin(2x-t)$

$$\partial_t u_3 - \partial_x^3 u_3 = -c_3 \cos(2x-t) + 8c_3 \cos(2x-t)$$

$$= +7c_3 \cos(2x-t) = \cos(2x-t)$$

$+7c_3 = 1$ so $c_3 = +\dfrac{1}{7}$

$$u_3 = c_3 \sin(2x-t)$$

$$\partial_x u_3 = 2c_3 \cos(2x-t)$$

$$\partial_x^2 u_3 = -4c_3 \sin(2x-t)$$

$$\boxed{u_3 = \frac{1}{7}\sin(2x-t)}$$

$$u = u_1 + u_2 + u_3$$

$$= -\frac{1}{60}x^5 - \frac{1}{2}t^2 + \frac{1}{7}\sin(2x-t)$$

$\underline{\underline{2}}$ $\quad \vec{F}(x,y) = (x, y) \quad$: D is unit disc

gradient

a/ $\displaystyle\int_D \nabla \cdot \vec{F}(x,y)\, dx\, dy$

dot product / inner product

$\nabla = (\partial_x, \partial_y)$

$\vec{F} = (x, y)$

$\nabla \cdot F = \partial_x x + \partial_y y = 1 + 1 = 2$

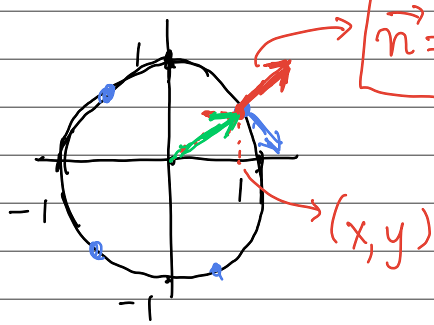
$$\int_D 2\, dx\, dy = 2 \cdot \underbrace{\text{Area of } D}_{\text{area of unit circle}} = 2\pi$$

b/ $\displaystyle\int_{\partial D} \vec{F}(x,y) \cdot \boxed{\vec{n}(x,y)}\, dS = \int_{\partial D} 1\, dS = \text{length of } \partial D$

$= 2\pi \cdot 1 = 2\pi$

unit normal vector outward

$F(x,y) = (x,y)$



$\boxed{\vec{n} = (x, y)}$

$(x,y) \cdot (x,y)$

$= x^2 + y^2$ on ∂D

$= \underline{1}$

B : linear

A $\quad$ C

3/ $(1+u_y^2)u_{xx} - \boxed{2u_x u_y u_{xy}} + (1+u_x^2)u_{yy} = 0$

second derivative $\longrightarrow$ order is 2

quasi-linear

not linear

$\boxed{\partial_t u} - \boxed{\partial_x^2 u} - u^3 = 0$: semi-linear

linear in the partial

1/ $\underbrace{\partial_t^2 u} - \underbrace{4\partial_x^2 u} = \sin^4(xy)$: $\boxed{\text{order } 2}$

linear in the partial $\qquad$ linear