$* \underline{\underline{2}} *$ $\quad \vec{F}(x,y) = (x, y)$

D unit disc , ∂D: unit circle

a/ $\displaystyle\iint_D \nabla \cdot \vec{F}(x,y)\,dx\,dy$

dot product / inner product

$\nabla = (\partial_x \,,\, \partial_y)$

$\vec{F} = (x \,,\, y)$

$\nabla \cdot \vec{F} = \underbrace{\partial_x x} + \underbrace{\partial_y \cdot y} = 1 + 1 = 2$

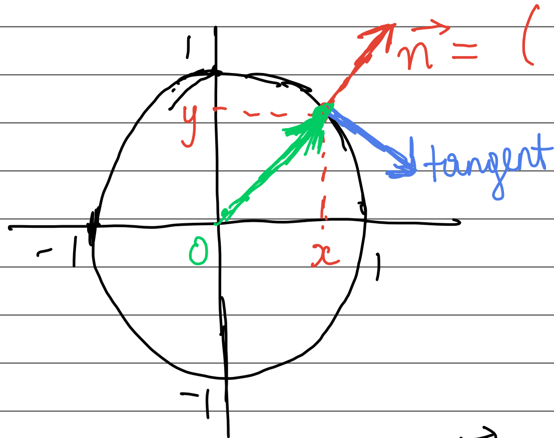
$\displaystyle\int_D 2\,dx\,dy = 2 \underbrace{\text{Area of } D}_{\text{unit disc}} = 2\pi$

Area of unit disc $= \pi \cdot 1^2 = \pi$

b/ $\displaystyle\int_{\partial D} \vec{F} \cdot \vec{n}(x,y)\,ds$

$\vec{F} = (x, y)$

$\vec{n}(x,y) \overset{?}{=}$ normal vector outward (unit)



$\vec{n} = (x, y)$

green = red (x, y)

(x, y)

$$\int_{\partial D} \vec{F} \cdot \vec{n} \, ds$$

$\vec{F} = (x, y)$

$\vec{n} = (x, y)$

$\vec{F} \cdot \vec{n} = x^2 + y^2$

on ∂D: $x^2 + y^2 = 1$

unit circle

$$\int_{\partial D} 1 \, dS = \text{length of } (\partial D)$$

$= \text{length of (unit circle)}$

$= 2\pi \cdot 1 = 2\pi$

4/ $\partial_t u - \partial_x^3 u = x^2 - t + \cos(2x-t)$

differential operator : $L(u) = \partial_t u - \partial_x^3 u$

① $L(u) = x^2$: find u_1

② $L(u) = -t$: find u_2

③ $L(u) = \cos(2x-t)$: find u_3

$$u = u_1 + u_2 + u_3$$

① Solve : $\partial_t u - \partial_x^3 u = x^2$

Guess : $u_1 = C_1 x^5$

$$\partial_t u_1 - \underbrace{\partial_x^3 u_1} = 0 - 60 C_1 x^2 = x^2$$

therefore $C_1 = -\frac{1}{60}$

$$\boxed{u_1 = -\frac{1}{60} x^2}$$

② Solve: $L(u) = -t$

$$\partial_t u - \partial_x^3 u = -t$$

Guess: $u_2 = C_2 t^2$

$$\partial_t u_2 - \partial_x^3 u_2 = 2C_2 t = -t$$

therefore $C_2 = -\frac{1}{2}$

$$\boxed{u_2 = -\frac{1}{2}t^2}$$

③ Solve $L(u) = \cos(2x-t)$

$$\partial_t u - \partial_x^3 u = \cos(2x-t)$$

$$\boxed{u_3 = \frac{1}{7}\sin(2x-t)}$$

Guess: $u_3 = C_3 \sin(2x-t)$

$$\partial_t u_3 - \partial_x^3 u_3 = -C_3\cos(2x-t) - \left(-8C_3\cos(2x-t)\right)$$
$$= -C_3\cos(2x-t) + 8C_3\cos(2x-t)$$
$$= 7C_3\cos(2x-t)$$
$$= \cos(2x-t)$$

$$\boxed{\begin{aligned} \partial_x u_3 &= 2C_3\cos(2x-t) \\ \partial_x^2 u_3 &= -4C_3\sin(2x-t) \\ \partial_x^3 u_3 &= -8C_3\cos(2x-t) \end{aligned}}$$

Therefore $C_3 = \frac{1}{7}$

4/

$$u = u_1 + u_2 + u_3$$

$$u = -\frac{1}{60}x^2 - \frac{1}{2}t^2 + \frac{1}{7}\sin(2x - t)$$

* 3/ a/ $\partial_t^{2} u - 4\partial_x^2 u = \sin^4(xy)$ u? not your $\underline{u}$!!

order : " derivative " : $\boxed{\text{order } 2}$

terms with derivative : $\underline{1}\cdot\partial_t^2 u$, $\underline{-4}\partial_x^2 u$: linear

$\boxed{\text{" u " term : (no derivative)}}$: no

b/ $\partial_t u - \partial_x^2 u - u^3 = 0$ ⟹ semi - linear

derivative ⟹ linear

u : power three ⟶ not linear

c/ $(1+u_y^2)u_{xx} - 2\boxed{u_x u_y u_{xy}} + (1+u_x^2)u_{yy} = 0$

second deri ⟹ order = 2

quasi

1/ $f = \ln(x^2+y^2)$

$$\Delta = \partial_x^2 f + \partial_y^2 f = 0$$

- $\partial_x f = \dfrac{2x}{(x^2+y^2)}$

$$\partial_x^2 f = \frac{2(x^2+y^2) - 2x \cdot (2x)}{(x^2+y^2)^2} = \frac{2y^2 - 2x^2}{(x^2+y^2)^2}$$

$$\partial_{\cancel{x}y}^2 f = \frac{2x^2 - 2y^2}{(x^2+y^2)^2}$$