3 $x^2 \partial_x u + y^2 \partial_y u = u^2$

$u(x, 2x) = -2x$

Step 1: $a = x^2, \quad b = y^2, \quad c = u^2$

Step 2: $x_0(s) = s \quad , y_0(s) = 2s \quad , u_0(s) = -2s$

Step 3:
$$\begin{cases} \partial_t x = x^2 , & x_0(s) = s \quad (1) \\ \partial_t y = y^2 , & y_0(s) = 2s \quad (2) \\ \partial_t u = u^2 , & u_0(s) = -2s \quad (3) \end{cases}$$

$\frac{d}{dt} x = x^2 \quad : \quad \frac{1}{x^2} dx = dt \to t$

integrate 0 to t

$\frac{d}{dt} f = f^2$

$-\frac{1}{x}$

$$-\frac{1}{x(t,s)} - \left(-\frac{1}{x(0,s)}\right) = t$$

$$\boxed{-\frac{1}{x} + \frac{1}{s} = t}$$

$$\frac{1}{s} - t = \frac{1}{x}$$

$$\frac{1-ts}{s} = \frac{1}{x}$$

$$\boxed{x = \frac{s}{1-ts}}$$

$$y = \frac{2s}{1-2ts}$$

$$u = \frac{-2s}{1+2ts}$$

$$u = \frac{-2s}{1+2ts}, \quad x = \frac{s}{1-ts}, \quad y = \frac{2s}{1-2ts}$$

$$1 - ts = \frac{s}{x} \Rightarrow ts = 1 - \frac{s}{x}$$

$$y = \frac{2s}{1-2\left(1-\frac{s}{x}\right)} = \frac{2s}{-1+\frac{2s}{x}} = \frac{2sx}{2s-x}$$

$$y(2s-x) = 2sx$$

$$2sy - yx = 2sx$$

$$2s(y-x) = yx \Rightarrow \boxed{s = \frac{yx}{2(y-x)}}$$

$$t = \frac{1}{s} - \frac{1}{x} = \frac{2(y-x)}{yx} - \frac{1}{x} = \frac{2(y-x)-y}{yx} = \frac{y-2x}{yx}$$

$$u = \frac{-2s}{1+2ts} = \frac{-\cancel{2}\left(\frac{yx}{\cancel{2}(y-x)}\right)}{1+\cancel{2}\left(\frac{y-2x}{\cancel{yx}}\right)\left(\frac{\cancel{yx}}{\cancel{2}(y-x)}\right)}$$

$$= \frac{\frac{-xy}{y-x}}{1+\frac{y-2x}{y-x}} = \frac{-xy}{(y-x)+(y-2x)} = \frac{-xy}{-3x+2y}$$

$(1,2,-2)$

$$x_0(s) = s, \quad y_0(s) = 2s, \quad u_0(s) = -2s$$
$$\quad \| \qquad\qquad \| \qquad\qquad \|$$
$$\quad 1 \qquad\qquad 2 \qquad\quad -2$$

$$\boxed{s = 1}$$

At $t=0$, $s=1$:
$$\boxed{\begin{aligned} a &= [x_0(1)]^2 = 1^2 = 1 \\ b &= [y_0(1)]^2 = (2)^2 = 4 \end{aligned}}$$
$$c = [u_0(1)]^2 = (-2)^2 = 4$$

$x_0' = 1$, $y_0' = 2$

$$J = \begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} = -2 \neq 0:$$

<u>transversality</u> condition holds

$\Rightarrow$ unique solution

near $(1,2,-2)$.

* 1 *

$$x\,\partial_x u - y\,\partial_y u = -u$$

$$u(x,y) = 2 \quad \text{on } \{x^2+y^2=1\}$$

$$P\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 2\right)$$

Step1: $a = x, \quad b = -y, \quad c = -u$

Step2: initial para :

$x_0(s) = \cos s, \quad y_0(s) = \sin s, \quad u_0(s) = 2$

$0 \leq s \leq 2\pi$

$$(\cos s, \sin s, 2) = P\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 2\right)$$

$\cos s = -\frac{\sqrt{2}}{2}$

$\sin s = \frac{\sqrt{2}}{2}$ $\Longrightarrow$ $\boxed{s = \frac{3\pi}{4}}$

$a = x_0(s) = \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$b = -y_0(s) = -\sin\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$$J = \begin{vmatrix} a & b \\ x' & y' \end{vmatrix} = \begin{vmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{vmatrix} \boxed{= 0}$$

$x' = -\sin s, \quad x'\left(\frac{3\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$y' = \cos s, \quad y'\left(\frac{3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

transversality condition fails. no soln / infinitely many soln = parallel

Check if $(\underline{a}, \underline{b}, \underline{c})\Big|_{t=0,\ s=\frac{3\pi}{4}}$

is parallel to $(x_0', y_0', u_0')\Big|_{t=0,\ s=\frac{3\pi}{4}}$

$c = -u_0\left(\frac{3\pi}{4}\right) = -2.$

$u_0' = 0$

$(a, b, c) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -2\right)$

$(x_0', y_0', u_0') = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)$

<u>NOT</u> parallel $\rightsquigarrow$ not the case infinite soln

$\longrightarrow$ <u>NO</u> soln.