Goldman Sachs Asset Management: Commitment Strategy Optimization

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Illiquid Assets

What are illiquid assets?

- Liquid assets can be quickly bought and sold without losing value.
- Whereas illiquid assets cannot be resold quickly without a loss in overall value.
- Example: Stocks are liquid and real estate is illiquid.
- Illiquid assets have played an increasingly large role in the portfolios of institutional investors.

Background to the Problem

- In 2006, these alternative assets for US life insurers' had a total value of $8.6 billion, however in 2021, these accounted for $122 billion.[1]
- Unlike liquid assets such as stocks, investing in illiquid assets is typically done through limited partnership funds, which introduces additional implementation challenges.
- Some institutional investors, like insurance companies, are tightly constrained by regulations, and need to target specific asset allocations.
- When investing in illiquid assets, the investor loses control over the timing of the distribution of capital while having some limited control over the timing of capital contributions.

[1] Based on S&P Global SNL Financial data, GSAM

The Problem

We want to define a multiyear commitment plan to achieve the desired asset exposure for a client.

- How much should the client commit each year in order to reach the target net asset value as closely as possible?
- What impact will stochastic market growth rates have on optimal client investment strategies?

Fund Dynamics

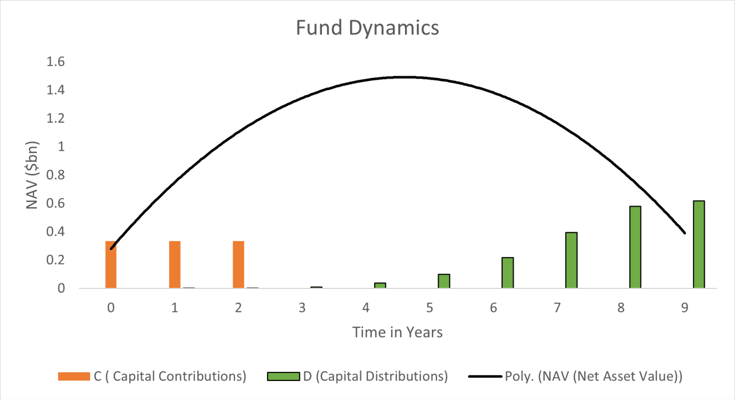


Figure:
C: The Limited Partner (LP) contributes capital to General Partner (GP).
D: The GP distributes capital to the LP.

The Yale Model: Key Variables

Variable	Name
$C_{(t)}^{(v)}$	Capital contributions
$D_{(t)}^{(v)}$	Capital distributions
$\mathrm{NAV}_{(t)}^{(v)}$	Net Asset Value
$\mathrm{PIC}_{(t)}^{(v)}$	Paid in capital for year t (\$)
$\mathrm{RD}_{(t)}$	Rate of distribution in year t (%)

Time (t) refers to the time at which we review the fund.
Vintage (v) refers to the time at which the fund was created.

The Yale Model: Key Parameters

The model is calibrated using the following parameters:

Variable	Name
$\mathrm{RC}_{(t)}$	Rate of contribution in year t (%)
$\mathrm{CC}^{(v)}$	Capital commitments ($)
L	Life of fund
B	Bow
$G_{(t)}^{(v)}$	**Random** Annual Growth Rate (%)
Y	Yield (%)

The Yale Model

The dynamical equations now extended to account for vintages and stochastic growth become:

$$\mathrm{NAV}_{(t)}^{(v)} = \left(\mathrm{NAV}_{(t-1)}^{(v)}\left(1 + G_{(t)}^{(v)}\right)\right) + C_{(t)}^{(v)} - D_{(t)}^{(v)}$$

where

$$C_{(t)}^{(v)} = \mathrm{RC}_{(t-v)}\left(\mathrm{CC}^{(v)} - \sum_{i=v}^{t-1} C_{(i)}^{(v)}\right)$$

$$D_{(t)}^{(v)} = \mathrm{RD}_{(t-v)}\left(\mathrm{NAV}_{(t-1)}^{(v)}\left(1 + G_{(t)}^{(v)}\right)\right)$$

The total exposure at time t to private equities is:

$$\mathrm{NAV}_{(t)} = \sum_{v=0}^{t} \mathrm{NAV}_{(t)}^{(v)}$$

Yale Model Results: Multi-Vintage, no randomness

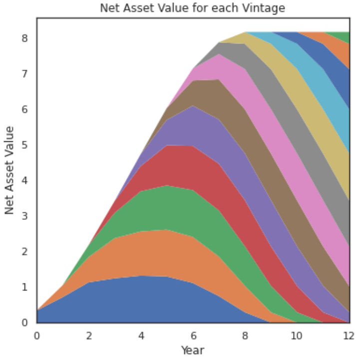


Figure: NAVs form a stack, which gives the total NAV.

A Naive Heuristic Approach

- Our target is T, the steady state value for the deterministic model with all the $\mathrm{CC}^{(v)} = 1$.
- At each time t, we can choose $\mathrm{CC}^{(t+1)}$, which is the commitment for the vintage that starts the following year.
- We naively assume a simple form for $\mathrm{CC}^{(t+1)}$:

$$\mathrm{CC}^{(t+1)} = \max\left\{1 + \alpha\left(\mathrm{NAV}_{(t)} - T\right) + \beta, 0\right\} \tag{1}$$

where α and β are parameters that we optimize over.

The Objective Function

- First, we pick year 20 as a reference year and compute all the quantities at this year.
- Next, we consider some number of realizations of the stochastic process, typically 1000, to account for the noisy nature of the problem.
- Then, we define the objective function as the mean square error:

$$L\left(\mathrm{NAV}_{(20)}\right) = \mathbb{E}\left[(\mathrm{NAV}_{(20)} - T)^2\right] \quad (2)$$

- Here $\mathrm{NAV}_{(20)}$ is the vector of $\mathrm{NAV}_{(20)}$ across all the realizations, and T is the vectorized target.

Results with the Naive Approach

- No control ($\alpha = \beta = 0$) gives $L \approx 40$.
- Varying α and β gives:

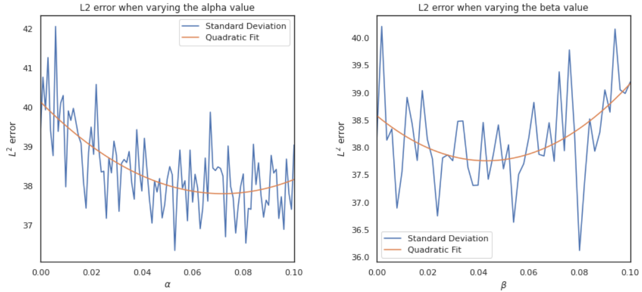


Figure: Varying (a) α and (b) β results in varying errors. Optimum occurs near $\alpha \approx 0.06$ and $\beta \approx 0.03$, which gives $L \approx 38$.

- Hence, naive approach gives 5% improvement over no control.

A More Complex Heuristic Approach

- 5% is not 0% improvement, but it also isn't very good.
- So consider a more sophisticated approach, defining

$$\mathrm{CC}^{(t+1)} = 1 + \langle S, \mathrm{NAV}_A \rangle + \beta. \qquad (3)$$

- NAV_A is a vector comprised of $\mathrm{NAV}_{(s)}$ for s ranging from $t - L + 1$ to t, giving the net asset values for the previous L years.
- S is a strategy vector with components that we want to find, and β is another constant additive factor that we also want to find.

Gradient Descent

- To find S and β, use gradient descent, a common technique in convex optimization.
- For a function F, to find the minimum, we iteratively perform the update:

$$x_{n+1} = x_n - \gamma \nabla F(x_n) \tag{4}$$

  In our case, F is the loss function we are using
- Here x_n is a sequence that hopefully converges to the minimum x^* and γ is a step size parameter.
- To compute the gradient, we use a centered finite difference method.
- Additionally, because of the stochasticity, we place limiters on the gradient so that the updates are not too large.

Gradient Descent Convergence

Gradient descent is guaranteed to converge for convex functions. Without noise, we believe that we would have convexity, based on the curve fits in Fig. 3. However, with the added noise, convergence is difficult.

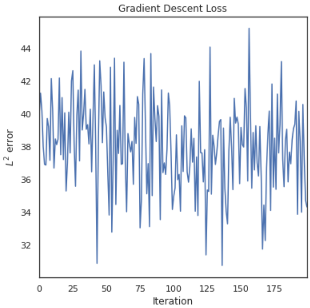


Figure: Loss for gradient descent iterations

Comparison with No Control

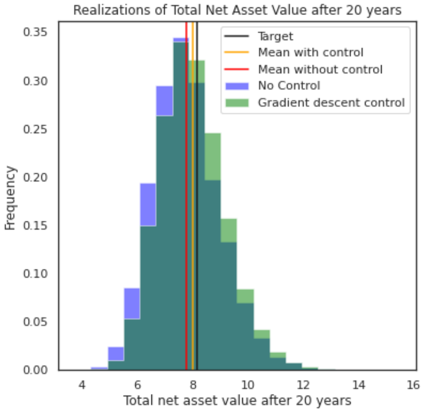


Figure: Comparison of gradient descent derived control with no control. We see that we get a mean much closer to the target value.

Basic Framework

Recasting the problem in a more standard notation, consider the following stochastic control problem:

$$\min_{u \in \mathcal{U}} J(t, x_t, v_t) = \min_{u \in \mathcal{U}} \mathbb{E} \left\{ \sum_{s=t}^{T} f_0(s, X_s, u_s(X_s, V_s) \right\} \tag{5}$$

subject to the discrete-time state equation

$$X_{t+1} = f_1(X_t, u_t, v_t), \quad X(t=0) = X_0 \in \mathbb{R} \tag{6}$$

Bellman's Principle of Optimality

The central tool in solving this optimization problem is Bellman's dynamic programming equation

$$J\left(t-1, x_{t-1}, v_{t-1}\right) = \min_{u_{t-1} \in \mathcal{U}} \left\{ f_0\left(t-1, x_{t-1}, u_{t-1}\right) + \mathbb{E}\left\{ J(t, x_t, v_t) \right\} \right\}$$

This equation is difficult to solve when the state x_t is high-dimensional (curse of dimensionality).

A Dynamic Program Applied to a Reduced Model

We solve the following stochastic control problem

$$\min_{CC} \mathbb{E} \left\{ \sum_{t=0}^{H-1} \left(NAV_{(t)} - T \right)^2 \right\}$$

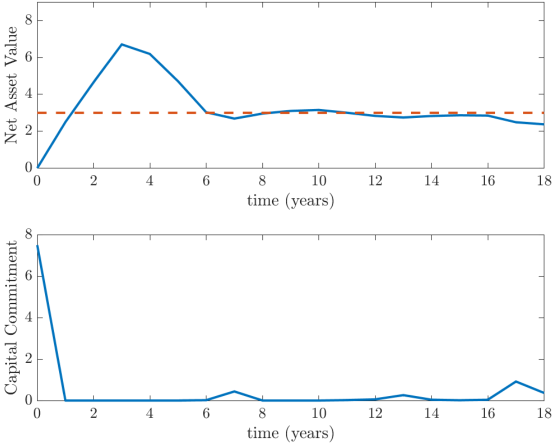
subject to a reduced model of the form

$$NAV_{t+1} = f\left(NAV_t, CC_t, CC_{t-1}, CC_{t-2}, W_{t+1} \right)$$

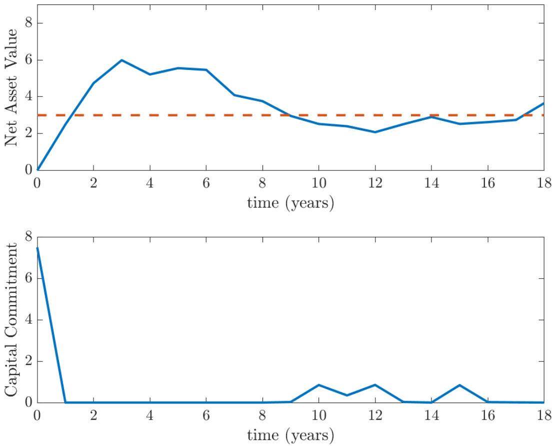
where the noise is Gaussian with specified means and variances.

Results of the Program

Setting the target $T = 3$, we observe the following result.



Another Result



Summary

This week, we investigated the stochastic multi-vintage Yale model, and we investigated a number of ways to perform control, including:

- Naive linear control
- Gradient descent optimized control
- Dynamic programming techniques

Future Work

Many things to work on:

- How do strategies change if we pick different RC?
- Does using the geometric mean work better?
- Better gradient descent convergence, maybe with a different cost function?
- Does the strategy depend on the distribution of the noise terms? What if the noise is normal inverse Gaussian distributed instead of normal?
- Will reinforcement learning work?
- Can we generalize to the multi-asset setting?

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