Solution

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Quiz 1

Math 9B, Winter 2026

1. Evaluate the indefinite integral

$$\int \left(2\sin(x) + 3x^4 - e^x - \frac{4}{x} + 5\right) dx$$

$$\boxed{2\cos x + \frac{3}{5}x^5 - e^x - 4\ln|x| + 5x + C}$$

2. Find $f(x)$ described by the initial value problem:

$$f''(x) = 3x,\ f'(0) = 1,\ f(0) = 2$$

$$f'(x) = \int 3x\,dx = \frac{3x^2}{2} + C$$

$$1 = \frac{3(0)^2}{2} + C \Rightarrow C = 1$$

$$f'(x) = \frac{3x^2}{2} + 1$$

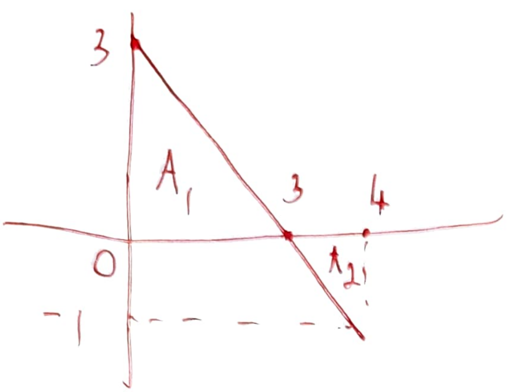
$$f(x) = \int\left(\frac{3x^2}{2} + 1\right) dx = \frac{3x^3}{2\,3} + x + C = \frac{x^3}{2} + x + C$$

$$2 = \frac{0^3}{2} + 0 + C \Rightarrow C = 2$$

$$\boxed{f(x) = \frac{x^3}{2} + x + 2}$$

3. Sketch the graph of $f(x) = 3 - x$ on the interval $[0, 4]$ and use it to evaluate

$$\int_0^4 (3 - x)dx$$



$$A_1 = \frac{1}{2}(3)(3) = \frac{9}{2}$$

$$A_2 = \frac{1}{2}(1)(1) = \frac{1}{2}$$

$$A_1 - A_2 = \frac{9}{2} - \frac{1}{2} = 4$$

4. Using the Fundamental Theorem of Calculus, evaluate

$$\int_0^1 (x + 2)^2 dx$$

$$= \int_0^1 (x^2 + 4x + 4)dx = \frac{x^3}{3} + 2x^2 + 4x \Big|_0^1$$

$$= \frac{1}{3} + 2(1) + 4(1)$$

$$- 0$$

$$= \frac{19}{3}$$

5. Evaluate the integral using only the definition (Riemann Sums). Hint: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\int_0^1 x\,dx \qquad f(x) = x$$

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$x^* = 0 + \frac{i}{n} = \frac{i}{n}$$

$$f(x^*) = f\left(\frac{i}{n}\right) = \frac{i}{n}$$

$$\int_0^1 x\,dx = \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(x^*)\Delta x\right)$$

$$= \lim_{n\to\infty}\left(\sum_{i=1}^{n} \frac{i}{n}\cdot\frac{1}{n}\right) = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{i}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty} \frac{1}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= \frac{1}{2}.$$