Math 9A Quiz 1 Solutions

Question 1

Consider the statements:

1. If $f(c) = 0$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = 0$.
2. If $g(c) = 0$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)}$ does not exist.

We must determine which option is correct.

Answer: (d) Neither (1) nor (2) is always true

Why statement (1) is not always true

Statement (1) says:

$$f(c) = 0 \quad \Longrightarrow \quad \lim_{x \to c} \frac{f(x)}{g(x)} = 0.$$

This is **not always true**. A counterexample is:

$$f(x) = x, \qquad g(x) = x, \qquad c = 0.$$

Then

$$f(0) = 0,$$

so the hypothesis is satisfied. But

$$\frac{f(x)}{g(x)} = \frac{x}{x} = 1 \qquad (x \neq 0),$$

and therefore

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = 1 \neq 0.$$

So statement (1) is false.

Why statement (2) is not always true

Statement (2) says:

$$g(c) = 0 \quad \Longrightarrow \quad \lim_{x \to c} \frac{f(x)}{g(x)} \text{ does not exist.}$$

This is also **not always true**. A counterexample is:

$$f(x) = x^2, \qquad g(x) = x, \qquad c = 0.$$

Then

$$g(0) = 0,$$

so the hypothesis is satisfied. But for $x \neq 0$,

$$\frac{f(x)}{g(x)} = \frac{x^2}{x} = x.$$

Hence

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} x = 0,$$

which **does exist**. So statement (2) is false.

Conclusion

Since statement (1) is not always true and statement (2) is not always true, the correct choice is

$$\boxed{(d) \text{ Neither (1) nor (2) is always true.}}$$

Question 2

Draw a function $y = f(x)$ satisfying:

- Domain of f is $[-2, -1) \cup (-1, 3)$
- $\lim_{x \to -1} f(x) = 3$
- $f(0) = 2$
- $\lim_{x \to 2^+} f(x) = -2$
- $f(2) = 1$
- $\lim_{x \to 2^-} f(x) = 0$

One possible function

A simple example is

$$f(x) = \begin{cases} x + 4, & -2 \le x < -1, \\ 2 - x, & -1 < x < 2, \\ 1, & x = 2, \\ -2, & 2 < x < 3. \end{cases}$$

Why this works

1. Domain The function is defined on

$$[-2, -1) \cup (-1, 2) \cup \{2\} \cup (2, 3) = [-2, -1) \cup (-1, 3),$$

which matches the required domain.

2. Limit as $x \to -1$ For $x < -1$, we use $f(x) = x + 4$, so

$$\lim_{x \to -1^-} f(x) = (-1) + 4 = 3.$$

For $-1 < x < 2$, we use $f(x) = 2 - x$, so

$$\lim_{x \to -1^+} f(x) = 2 - (-1) = 3.$$

Since both one-sided limits equal 3,

$$\lim_{x \to -1} f(x) = 3.$$

3. **Value at $x = 0$** Since 0 lies in the interval $(-1, 2)$, we use $f(x) = 2 - x$:

$$f(0) = 2 - 0 = 2.$$

4. **Left-hand limit at $x = 2$** For $x < 2$, we use $f(x) = 2 - x$, so

$$\lim_{x \to 2^-} f(x) = 2 - 2 = 0.$$

5. **Right-hand limit at $x = 2$** For $x > 2$, we use $f(x) = -2$, so

$$\lim_{x \to 2^+} f(x) = -2.$$

6. **Function value at $x = 2$** We define

$$f(2) = 1,$$

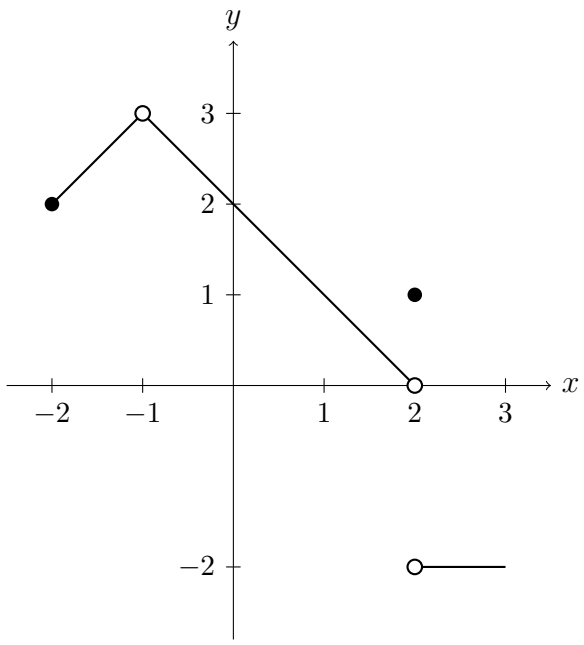
which matches the condition.
So this function satisfies every requirement.

Graph description

The graph should have:

- a line segment from $(-2, 2)$ up to an *open circle* at $(-1, 3)$,
- another line segment from an *open circle* at $(-1, 3)$ down to an *open circle* at $(2, 0)$,
- a *filled dot* at $(2, 1)$,
- a horizontal line $y = -2$ for $2 < x < 3$, with an *open circle* at $(2, -2)$.

Graph of the function



Final answer for Question 2

One valid answer is

$$\boxed{f(x) = \begin{cases} x + 4, & -2 \le x < -1, \\ 2 - x, & -1 < x < 2, \\ 1, & x = 2, \\ -2, & 2 < x < 3. \end{cases}}$$