Math 9A Quiz 2v1 Solutions

Problem 1

We want to arrange the following three quantities from least to greatest:

$$g'(1), \qquad g'(3), \qquad \frac{g(3)-g(0.5)}{3-0.5}.$$

Step 1: Compare $g'(1)$ and $g'(3)$

From the graph, the curve is decreasing at $x = 1$, so the tangent slope there is negative. Thus

$$g'(1) < 0.$$

At $x = 3$, the curve is increasing very steeply, so the tangent slope there is positive and fairly large. Thus

$$g'(3) > 0.$$

Therefore,

$$g'(1) < g'(3).$$

Step 2: Estimate the secant slope

The quantity

$$\frac{g(3)-g(0.5)}{3-0.5}$$

is the slope of the secant line from $x = 0.5$ to $x = 3$.

From the graph, $g(0.5)$ is a little above 0, while $g(3)$ is about 0. So the numerator

$$g(3) - g(0.5)$$

is slightly negative. Since the denominator is positive,

$$3 - 0.5 = 2.5 > 0,$$

the secant slope is also slightly negative.

So the secant slope is negative, but not as negative as $g'(1)$, because at $x = 1$ the graph is decreasing more sharply than the average decrease over the whole interval from 0.5 to 3.

Hence,

$$g'(1) < \frac{g(3)-g(0.5)}{3-0.5} < g'(3).$$

Conclusion

The correct order from least to greatest is

$$g'(1) < \frac{g(3) - g(0.5)}{3 - 0.5} < g'(3).$$

So the correct choice is

$$\boxed{\text{(c)}}$$

Problem 2

Let

$$k(t) = \frac{t^3 e^t}{\sin t}.$$

Find the slope of the tangent line to $k(t)$ at $t = 1$.

The slope of the tangent line is $k'(1)$, so we first differentiate.

Method: Quotient Rule

Let

$$u(t) = t^3 e^t, \qquad v(t) = \sin t.$$

Then

$$k(t) = \frac{u(t)}{v(t)},$$

so by the quotient rule,

$$k'(t) = \frac{u'(t)v(t) - u(t)v'(t)}{(v(t))^2}.$$

Step 1: Differentiate $u(t) = t^3 e^t$

Use the product rule:

$$u'(t) = (t^3)' e^t + t^3 (e^t)'.$$

Thus

$$u'(t) = 3t^2 e^t + t^3 e^t = e^t(3t^2 + t^3).$$

Step 2: Differentiate $v(t) = \sin t$

$$v'(t) = \cos t.$$

Step 3: Apply the quotient rule

$$k'(t) = \frac{e^t(3t^2 + t^3)\sin t - t^3 e^t \cos t}{\sin^2 t}.$$

Factor out $t^2 e^t$:

$$k'(t) = \frac{t^2 e^t\big((3 + t)\sin t - t\cos t\big)}{\sin^2 t}.$$

Step 4: Evaluate at $t = 1$

$$k'(1) = \frac{1^2 e^1\big((3+1)\sin 1 - 1\cos 1\big)}{\sin^2 1}.$$

So

$$k'(1) = \frac{e(4\sin 1 - \cos 1)}{\sin^2 1}.$$

Therefore, the slope of the tangent line at $t = 1$ is

$$\boxed{\frac{e(4\sin 1 - \cos 1)}{\sin^2 1}}.$$