Math 9A Worksheet 2 Solutions

Problem 2: Graph Construction

We construct a function satisfying the following conditions:

- $f(-2) = 3$ and $\lim_{x\to -2} f(x) = 1$
- $f(-1) = 1$ and $\lim_{x\to -1} f(x) = 1$
- $f(1)$ undefined and $\lim_{x\to 1} f(x) = 0$
- $f(2) = -1$ and $\lim_{x\to 2} f(x)$ does not exist

Explanation

At $x = -2$

The limit equals 1 but the function value is 3. Therefore the graph must approach the point $(-2, 1)$ but the actual function value is plotted at

$$(-2, 3).$$

So we draw

- an open circle at $(-2, 1)$
- a filled circle at $(-2, 3)$

At $x = -1$

Both the limit and function value equal 1, so the graph is continuous at

$$(-1, 1).$$

At $x = 1$

The limit equals 0 but the function is undefined. So the graph approaches $(1, 0)$ but we draw an open circle there.

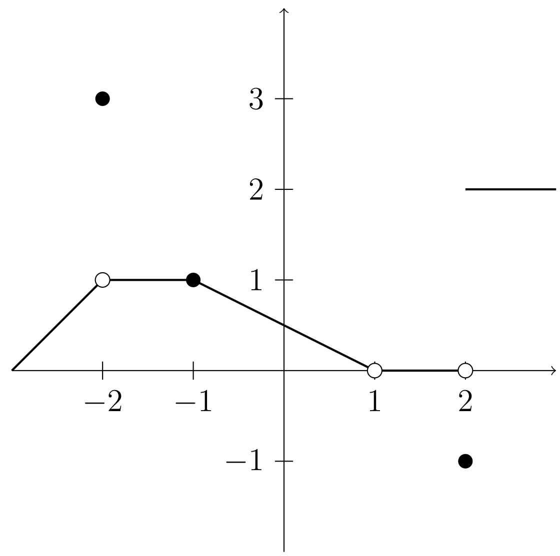
At $x = 2$

The function value is -1, but the limit does not exist. Thus the left and right limits must be different.

Example:

$$\lim_{x\to 2^-} f(x) = 0, \qquad \lim_{x\to 2^+} f(x) = 2.$$

Graph



Problem 4: Limits from Graphs

Two functions f and g are given graphically.

(a)

$$\lim_{x\to 0}(f(x)+2g(x))$$

Using limit laws:

$$\lim(f+2g)=\lim f+2\lim g$$

We read the values directly from the graphs.

(b)

$$\lim_{x\to 1}(f(x)g(x))$$

Using the product law:

$$\lim(fg)=(\lim f)(\lim g)$$

provided both limits exist.

(c)

$$\lim_{x\to 0} g(f(x))$$

First determine

$$\lim_{x\to 0} f(x)$$

from the graph of f. Then substitute that value into the function g.

(d)

$$\lim_{x\to 3^+}\frac{g(x)}{f(x)}$$

From the graphs:

- $g(x)$ approaches a negative value
- $f(x)$ approaches 0

Thus the quotient grows without bound.

$$\lim_{x\to 3^+}\frac{g(x)}{f(x)}=-\infty$$

Explanation

A limit equals $\pm\infty$ when the denominator approaches 0 while the numerator approaches a nonzero value.