TA: Khoi Vo
Student name:

Math 009B
March 11, 2026

Math 009B Worksheet 1 Solutions by Khoi

Question 1

Find $f(x)$ described by the given initial value problem.

1. $f'(x) = \sin x$ and $f(0) = 2$

Integrate:

$$f(x) = \int \sin x \, dx = -\cos x + C.$$

Use the initial condition:

$$f(0) = -\cos 0 + C = -1 + C = 2.$$

So

$$C = 3.$$

Therefore,

$$\boxed{f(x) = -\cos x + 3.}$$

2. $f'(x) = 5e^x$ and $f(0) = 10$

Integrate:

$$f(x) = \int 5e^x \, dx = 5e^x + C.$$

Use the initial condition:

$$f(0) = 5e^0 + C = 5 + C = 10.$$

So

$$C = 5.$$

Therefore,

$$\boxed{f(x) = 5e^x + 5.}$$

3. $f'(x) = 4x^3 - 3x^2$ **and** $f(-1) = 9$

Integrate:

$$f(x) = \int (4x^3 - 3x^2)\,dx = x^4 - x^3 + C.$$

Use the initial condition:

$$f(-1) = (-1)^4 - (-1)^3 + C = 1 - (-1) + C = 2 + C = 9.$$

So

$$C = 7.$$

Therefore,

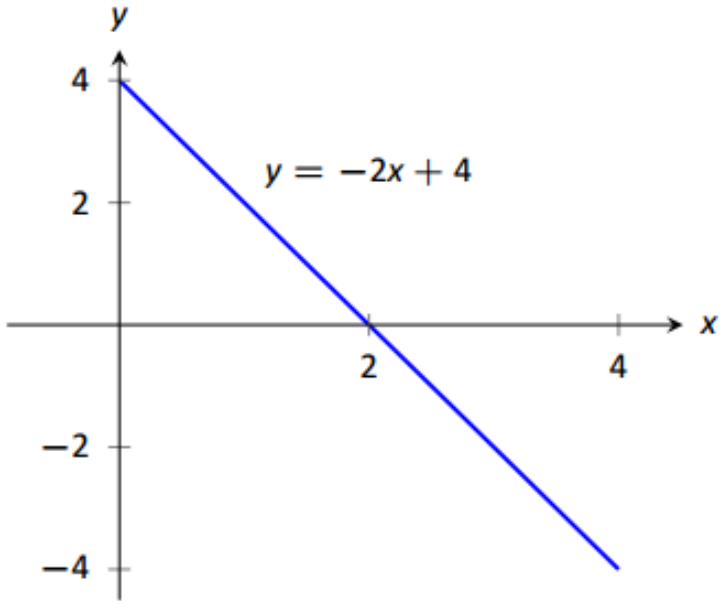
$$\boxed{f(x) = x^4 - x^3 + 7.}$$

Question 2

Using geometry, evaluate the following definite integrals for the line

$$y = -2x + 4.$$

This line crosses the y-axis at $(0, 4)$ and the x-axis at $(2, 0)$.



1. $\int_0^1 (-2x+4)\,dx$

From $x=0$ to $x=1$, the graph forms a trapezoid with heights 4 and 2 and width 1.

$$\text{Area} = \frac{1}{2}(4+2)(1) = 3.$$

Therefore,

$$\boxed{\int_0^1 (-2x+4)\,dx = 3.}$$

2. $\int_0^2 (-2x+4)\,dx$

From $x=0$ to $x=2$, the graph forms a triangle above the x-axis with base 2 and height 4.

$$\text{Area} = \frac{1}{2}(2)(4) = 4.$$

Therefore,

$$\boxed{\int_0^2 (-2x+4)\,dx = 4.}$$

3. $\int_1^3 (-2x+4)\,dx$

Break it into two parts.
From $x=1$ to $x=2$, there is a triangle above the x-axis with base 1 and height 2:

$$A_1 = \frac{1}{2}(1)(2) = 1.$$

From $x=2$ to $x=3$, there is a triangle below the x-axis with base 1 and height 2:

$$A_2 = -\frac{1}{2}(1)(2) = -1.$$

So

$$\int_1^3 (-2x+4)\,dx = 1 + (-1) = 0.$$

Therefore,

$$\boxed{\int_1^3 (-2x+4)\,dx = 0.}$$

4. $\int_0^3 (-2x+4)\,dx$

From $x=0$ to $x=2$, the area is 4. From $x=2$ to $x=3$, the area is a triangle below the axis with area -1.

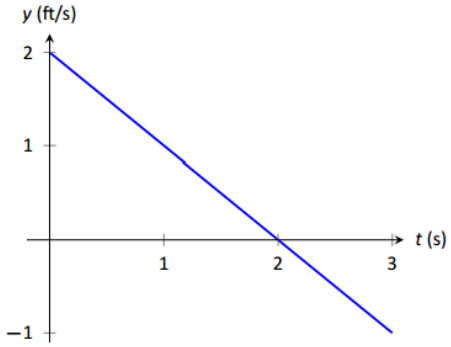
Thus,

$$\int_0^3 (-2x+4)\,dx = 4-1 = 3.$$

Therefore,

$$\boxed{\int_0^3 (-2x+4)\,dx = 3.}$$

Question 3



A graph of the velocity is given. From the graph, the velocity line passes through $(0,2)$ and $(3,-1)$, so

$$v(t) = 2-t \qquad \text{for } 0 \le t \le 3.$$

1. What is the object's maximum velocity?

Since the graph starts at $v(0)=2$ and decreases afterward, the maximum velocity is

$$\boxed{2 \text{ ft/s}.}$$

2. What is the object's maximum displacement?

Displacement increases while velocity is positive. Since

$$v(t) = 2-t = 0 \quad \text{when } t = 2,$$

the maximum displacement occurs at $t = 2$.

From $t = 0$ to $t = 2$, the graph forms a triangle with base 2 and height 2:

$$\text{Displacement} = \frac{1}{2}(2)(2) = 2.$$

Therefore, the maximum displacement is

$$\boxed{2 \text{ ft.}}$$

3. What is the object's total displacement on $[0, 3]$?

From $t = 0$ to $t = 2$, the positive area is

$$\frac{1}{2}(2)(2) = 2.$$

From $t = 2$ to $t = 3$, the negative area is

$$-\frac{1}{2}(1)(1) = -\frac{1}{2}.$$

So the total displacement is

$$2 - \frac{1}{2} = \frac{3}{2}.$$

Therefore,

$$\boxed{\frac{3}{2} \text{ ft.}}$$

Question 4

An object is thrown straight up with velocity

$$v(t) = -32t + 64$$

from a height of 48 feet.

1. What is the object's maximum velocity?

Since $v(t)$ is decreasing in t, its maximum occurs at $t = 0$:

$$v(0) = -32(0) + 64 = 64.$$

Therefore,

$$\boxed{64 \text{ ft/s.}}$$

2. What is the object's maximum displacement?

Displacement is the integral of velocity:

$$s(t) = \int (-32t + 64)\, dt = -16t^2 + 64t + C.$$

If displacement is measured from the initial position, then $s(0) = 0$, so $C = 0$. Thus

$$s(t) = -16t^2 + 64t.$$

Maximum displacement occurs when velocity is zero:

$$-32t + 64 = 0 \implies t = 2.$$

Then

$$s(2) = -16(2)^2 + 64(2) = -64 + 128 = 64.$$

Therefore, the maximum displacement is

$$\boxed{64 \text{ ft.}}$$

3. When does the object reach a height of 0?

The object starts at height 48, so height is

$$h(t) = 48 + s(t) = 48 - 16t^2 + 64t.$$

Set height equal to 0:

$$48 - 16t^2 + 64t = 0.$$

Divide by -16:

$$t^2 - 4t - 3 = 0.$$

Use the quadratic formula:

$$t = \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}.$$

Since time must be positive,

$$\boxed{t = 2 + \sqrt{7} \text{ seconds.}}$$

Question 5

Given:

$$\int_0^2 f(x)\,dx = 5, \qquad \int_0^3 f(x)\,dx = 7,$$
$$\int_0^2 g(x)\,dx = -3, \qquad \int_2^3 g(x)\,dx = 5.$$

1. $\int_0^2 (f(x) + g(x))\,dx$

By linearity,

$$\int_0^2 (f(x) + g(x))\,dx = \int_0^2 f(x)\,dx + \int_0^2 g(x)\,dx = 5 + (-3) = 2.$$

Therefore,

$$\boxed{2.}$$

2. $\int_0^3 (f(x) - g(x))\,dx$

First find

$$\int_0^3 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^3 g(x)\,dx = -3 + 5 = 2.$$

Then

$$\int_0^3 (f(x) - g(x))\,dx = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx = 7 - 2 = 5.$$

Therefore,

$$\boxed{5.}$$

3. $\int_2^3 (3f(x) + 2g(x))\,dx$

First find

$$\int_2^3 f(x)\,dx = \int_0^3 f(x)\,dx - \int_0^2 f(x)\,dx = 7 - 5 = 2.$$

So

$$\int_2^3 (3f(x) + 2g(x))\,dx = 3\int_2^3 f(x)\,dx + 2\int_2^3 g(x)\,dx = 3(2) + 2(5) = 16.$$

Therefore,

$$\boxed{16.}$$

4. Find nonzero values a, b such that $\int_0^3 (ag(x) + bg(x))\,dx = 0$

Factor out $g(x)$:

$$\int_0^3 (ag(x) + bg(x))\,dx = \int_0^3 (a+b)g(x)\,dx = (a+b)\int_0^3 g(x)\,dx.$$

We already found

$$\int_0^3 g(x)\,dx = 2.$$

Thus

$$(a+b)(2) = 0 \implies a + b = 0.$$

Any nonzero numbers with sum 0 will work. For example,

$$\boxed{a = 1, \quad b = -1.}$$