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Week 5 solution by Khoi

Math 009B
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Question 1

Evaluate

1. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$. Let

$$u = \sqrt{x}$$

Then

$$x = u^2, \qquad dx = 2u\, du$$

Substitute:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = \int \frac{e^u}{u}(2u\, du) = \int 2e^u\, du$$

$$= 2e^u + C$$

Substitute back $u = \sqrt{x}$:

$$\boxed{\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2e^{\sqrt{x}} + C}$$

2. $\int_1^3 \frac{1}{x-5} dx$ We know

$$\int \frac{1}{x-5} dx = \ln|x-5| + C$$

Thus

$$\int_1^3 \frac{1}{x-5} dx = \big[\ln|x-5|\big]_1^3$$

$$= \ln|3-5| - \ln|1-5|$$

$$= \ln 2 - \ln 4$$

$$= \ln\left(\frac{1}{2}\right)$$

$$\boxed{\int_1^3 \frac{1}{x-5} dx = -\ln 2}$$

Question 2

Evaluate

1. $\int x\sin(x)dx$. Use integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Let

$$u = x \qquad dv = \sin x\,dx$$

Then

$$du = dx \qquad v = -\cos x$$

Thus

$$\int x\sin x\,dx = x(-\cos x) - \int(-\cos x)\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

$$\boxed{\int x\sin x\,dx = -x\cos x + \sin x + C}$$

2. $\int_0^\pi x\sin(x)dx$ From part (1)

$$\int x\sin x\,dx = -x\cos x + \sin x$$

Evaluate from 0 to π:

$$\int_0^\pi x\sin x\,dx = [-x\cos x + \sin x]_0^\pi$$

At $x = \pi$:

$$-\pi\cos\pi + \sin\pi = -\pi(-1) + 0 = \pi$$

At $x = 0$:

$$-0\cos 0 + \sin 0 = 0$$

Therefore

$$\boxed{\int_0^{\pi} x\sin x\,dx = \pi}$$

Question 3

Evaluate

1. $\int \sin^6(x)\cos^5(x)dx$. Write

$$\cos^5 x = \cos^4 x\cos x$$

Then

$$\int \sin^6 x\cos^5 x\,dx = \int \sin^6 x\cos^4 x\cos x\,dx$$

Use

$$\cos^4 x = (1-\sin^2 x)^2$$

So

$$\int \sin^6 x(1-\sin^2 x)^2\cos x\,dx$$

Let

$$u = \sin x \qquad du = \cos x\,dx$$

Then

$$\int u^6(1-u^2)^2\,du$$

Expand

$$(1-u^2)^2 = 1 - 2u^2 + u^4$$

So

$$\int (u^6 - 2u^8 + u^{10})\,du$$

Integrate

$$= \frac{u^7}{7} - \frac{2u^9}{9} + \frac{u^{11}}{11} + C$$

Substitute $u = \sin x$

$$\boxed{\frac{\sin^7 x}{7} - \frac{2\sin^9 x}{9} + \frac{\sin^{11} x}{11} + C}$$

2. Evaluate $\int \sin^2(x)\cos^2(x)\,dx$

Use

$$\sin(2x) = 2\sin x\cos x$$

Thus

$$\sin^2 x\cos^2 x = \frac{1}{4}\sin^2(2x)$$

Then

$$\int \sin^2 x\cos^2 x\,dx = \frac{1}{4}\int \sin^2(2x)\,dx$$

Use

$$\sin^2(2x) = \frac{1-\cos 4x}{2}$$

So

$$\frac{1}{4}\int \frac{1-\cos 4x}{2}dx = \frac{1}{8}\int (1-\cos 4x)dx$$

$$= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C$$

$$\boxed{\frac{x}{8} - \frac{\sin 4x}{32} + C}$$

2. $\int \tan^3(x)\sec^4(x)dx$ Write

$$\sec^4 x = \sec^2 x \sec^2 x$$

Then

$$\int \tan^3 x \sec^4 x\, dx = \int \tan^3 x \sec^2 x \sec^2 x\, dx$$

Let

$$u = \tan x$$

$$du = \sec^2 x\, dx$$

Then

$$\sec^2 x = 1 + \tan^2 x = 1 + u^2$$

Thus

$$\int u^3(1+u^2)du$$

$$= \int (u^3 + u^5)du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + C$$

Substitute $u = \tan x$

$$\boxed{\frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C}$$