quiz 2 solution new

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quiz 2 solution

Solution

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Quiz 2

Math 9B, Winter 2026

1. Use substitution to evaluate the integral

$$\int (2x+1)e^{(x^2+x+1)}dx$$

$u = x^2+x+1$

$du = (2x+1)\,dx$

$$\int e^{u}(2x+1)\,dx = \int e^{u}\,du = e^{u} + C$$

$$= e^{x^2+x+1} + C$$

2. Use integration by parts to evaluate the integral

$$\int \cos(x)\ln(\sin(x))dx$$

$u = \ln(\sin x) \qquad v = \sin x$

$du = \frac{\cos x}{\sin x}dx \qquad dv = \cos x\,dx$

$$\ln(\sin x)\sin x - \int (\sin x)\frac{\cos x}{\sin x}dx$$

$$= \ln(\sin x)\sin x - \int \cos x\,dx$$

$$= \ln(\sin x)\sin x - \sin x + C.$$

3. Evaluate the integral

$$\int \sin^3(x)\cos^{26}(x)dx$$

"the odd power" is splitted:

$$\int \sin x \cdot \sin^2 x \cdot \cos^{26}(x)\,dx$$

$$= \int \sin x\,(1-\cos^2 x)\cos^{26}(x)\,dx$$

Let $u = \cos x$, $du = -\sin x\,dx$, $dx = \frac{du}{-\sin x}$

$$= \int \sin x\,(1-u^2)u^{26}\frac{du}{-\sin x}$$

$$= \int (1-u^2)u^{26}\frac{du}{(-1)} = \int (u^{26}-u^{28})\frac{du}{(-1)}$$

$$= \int (u^{28}-u^{26})\,du = \frac{u^{29}}{29} - \frac{u^{27}}{27} + C$$

$$= \frac{\cos^{29}(x)}{29} - \frac{\cos^{27}(x)}{27} + C$$

4. Use trigonometric substitution to evaluate the integral

$$\int \frac{1}{\sqrt{x^2+1}}dx$$

$x = \tan\theta$, $dx = \sec^2\theta\,d\theta$

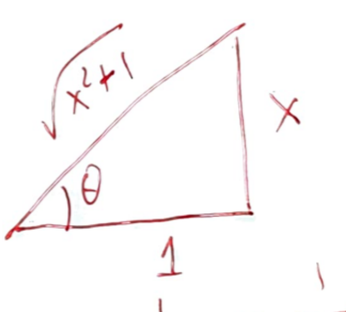
$$\sqrt{x^2+1} = \sqrt{\tan^2\theta+1} = \sqrt{\sec^2\theta} = \sec\theta$$

$$\int \frac{1}{\sec\theta}\sec^2\theta\,d\theta = \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

table in textbook

$$= \ln|x + \sqrt{x^2+1}| + C$$



$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\sqrt{x^2+1}}} = \sqrt{x^2+1}$$

5. Use partial fraction decomposition to evaluate the integral

$$\int \frac{3x+2}{x^2+x}dx$$

$$\frac{3x+2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

multiply by $x(x+1)$: $3x+2 = A(x+1) + Bx$

Choose $x=0$: $2 = A + B\cdot 0 = A$

Choose $x=-1$: $3(-1)+2 = A\cdot 0 + B(-1)$

$-1 = -B$

So $B = +1$

$$\frac{3x+2}{x(x+1)} = \frac{2}{x} + \frac{1}{x+1}$$

$$\int \frac{3x+2}{x^2+x}dx = \int\left(\frac{2}{x} + \frac{1}{x+1}\right)dx$$

$$= \int \frac{2}{x}dx + \int \frac{1}{x+1}dx$$

$$= 2\ln|x| + \ln|x+1| + C$$