MATH031 APPLIED LINEAR ALGEBRA
Quiz #1 Detailed Solutions

Prepared by Khoi Vo

Quiz source: MATH031 Applied Linear Algebra, Quiz #1, dated 04/09/2026.

Question 1

Perform elementary row operations to turn the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 \end{bmatrix}$$

into reduced row echelon form.

We will row reduce step by step and clearly indicate each row operation.

Step 1: Start with the given matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 \end{bmatrix}$$

The entry in row 1, column 1 is already a pivot candidate, so we use it to clear the entries below it.

Step 2: Eliminate the entries below the first pivot

Perform the row operations

$$R_2 \leftarrow R_2 - R_1, \qquad R_3 \leftarrow R_3 - 3R_1.$$

Then

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}.$$

Step 3: Eliminate the entry below the second pivot

Now the pivot in row 2, column 2 is already 1. Use it to eliminate the entry below it.

Perform

$$R_3 \leftarrow R_3 - R_2.$$

Then

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Step 4: Make the third pivot equal to 1

Currently the entry in row 3, column 3 is -1. To get a pivot of 1, multiply row 3 by -1:

$$R_3 \leftarrow -R_3.$$

Then

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

Step 5: Clear the entries above the third pivot

Now use the pivot in row 3, column 3 to eliminate the entries above it.

Perform

$$R_1 \leftarrow R_1 - R_3, \qquad R_2 \leftarrow R_2 + R_3.$$

Then

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

This matrix is now in reduced row echelon form.

Final Answer

Therefore, the reduced row echelon form of the matrix is

$$\boxed{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}}$$

Question 2

Suppose a linear system has augmented matrix in reduced row echelon form

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 5 & 1 & -2 \\ 0 & 0 & 1 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 1 & 4 & -1 & 6 \end{bmatrix}.$$

(a) Identify the lead variables and free variables. Is the system consistent?

Step 1: Determine how many variables there are

Since this is an augmented matrix with 7 columns total, the last column is the constants column. That means the system has

$$6 \text{ variables.}$$

Let the variables be

$$x_1, x_2, x_3, x_4, x_5, x_6.$$

Step 2: Identify pivot columns

The pivot columns are the columns containing the leading 1's. From the matrix, the leading 1's occur in:

$$\text{column 1, column 3, and column 4.}$$

So the **lead variables** (or basic variables) are

$$\boxed{x_1,\ x_3,\ x_4.}$$

The columns without pivots are columns 2, 5, and 6, so the **free variables** are

$$\boxed{x_2,\ x_5,\ x_6.}$$

Step 3: Check consistency

A system is inconsistent if a row of the form

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & b \end{bmatrix} \quad \text{with } b \neq 0$$

appears.

No such row appears here. Therefore the system is **consistent**.

So the answers are:

$$\boxed{\text{Lead variables: } x_1, x_3, x_4}$$

$$\boxed{\text{Free variables: } x_2, x_5, x_6}$$

$$\boxed{\text{The system is consistent.}}$$

(b) Describe the solution set in parametric vector form

Step 1: Write the system of equations

Translate the augmented matrix into equations:

From row 1:

$$x_1 + 2x_2 + 5x_5 + x_6 = -2.$$

From row 2:

$$x_3 - 3x_5 + 2x_6 = 0.$$

From row 3:

$$x_4 + 4x_5 - x_6 = 6.$$

Step 2: Solve for the lead variables

We solve each equation for its lead variable.

From the first equation:

$$x_1 = -2 - 2x_2 - 5x_5 - x_6.$$

From the second equation:

$$x_3 = 3x_5 - 2x_6.$$

From the third equation:

$$x_4 = 6 - 4x_5 + x_6.$$

Since x_2, x_5, x_6 are free, let

$$x_2 = s, \qquad x_5 = t, \qquad x_6 = u,$$

where $s, t, u \in \mathbb{R}$.

Then

$$x_1 = -2 - 2s - 5t - u,$$

$$x_3 = 3t - 2u,$$
$$x_4 = 6 - 4t + u.$$

So the general solution is

$$x_1 = -2-2s-5t-u, \qquad x_2 = s, \qquad x_3 = 3t-2u, \qquad x_4 = 6-4t+u, \qquad x_5 = t, \qquad x_6 = u.$$

Step 3: Write the solution in vector form

Thus

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2-2s-5t-u \\ s \\ 3t-2u \\ 6-4t+u \\ t \\ u \end{bmatrix}.$$

Separate the constant part and the parameter parts:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 6 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -5 \\ 0 \\ 3 \\ -4 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \qquad s, t, u \in \mathbb{R}.$$

Therefore, the solution set in parametric vector form is

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 6 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -5 \\ 0 \\ 3 \\ -4 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \qquad s, t, u \in \mathbb{R}.}$$

Copyright Notice

End of solutions